DOCUMENT RESUME

ED 373 990 SE 054 759

AUTHOR	Geddes, Dorothy; And Others
TITLE	Measurement in the Middle Grades: Curriculum and Evaluation Standards for School Mathematics. Addenda Series. Grades 5-8.
INSTITUTION	National Council of Teachers of Mathematics, Inc., Reston, Va.
REPORT NO	ISBN-0-87353-367-4
PUB DATE	94
NOTE	103p.
AVAILABLE FROM	National Council of Teachers of Mathematics, Inc., 1906 Association Drive, Reston, VA 22091-1593 (Stock No. 588, $12).
PUB TYPE	Guides - Classroom Use - Teaching Guides (For Teacher) (052)
EDRS PRICE	MF01 Plus Postage. PC Not Available from EDRS.
DESCRIPTORS	*Estimation (Mathematics); Intermediate Grades; Junior High Schools; *Mathematical Applications; Mathematics Curriculum; *Mathematics Instruction; *Measurement Techniques; Middle Schools; Worksheets
IDENTIFIERS	*Connections (Mathematics); Mathematics Activities; Measurement Student Activities; NCTM Curriculum and Evaluation Standards; Patterns (Mathematics)

ABSTRACT

This document presents activities that focus on learning measurement as it is used in the real world. The investigative approach used in these activities is designed to encourage middle-school students to explore ways of approaching real-life problems, to discover patterns, and to make and test hypotheses. These activity-oriented explorations are geared toward strengthening students' higher-order thinking skills and estimation. The 24 activities are clustered around three themes: estimating and measuring, discovering patterns and logical connections to create measurement formulas, and making connections through interdisciplinary measurement activities. Each cluster of investigations includes teacher notes with objectives and materials, easily duplicated activity sheets, and suggestions for integrating technology and alternative assessment. (MKR)

****
* Reproductions supplied by EDRS are the best that can be made *
* from the original document. *
****

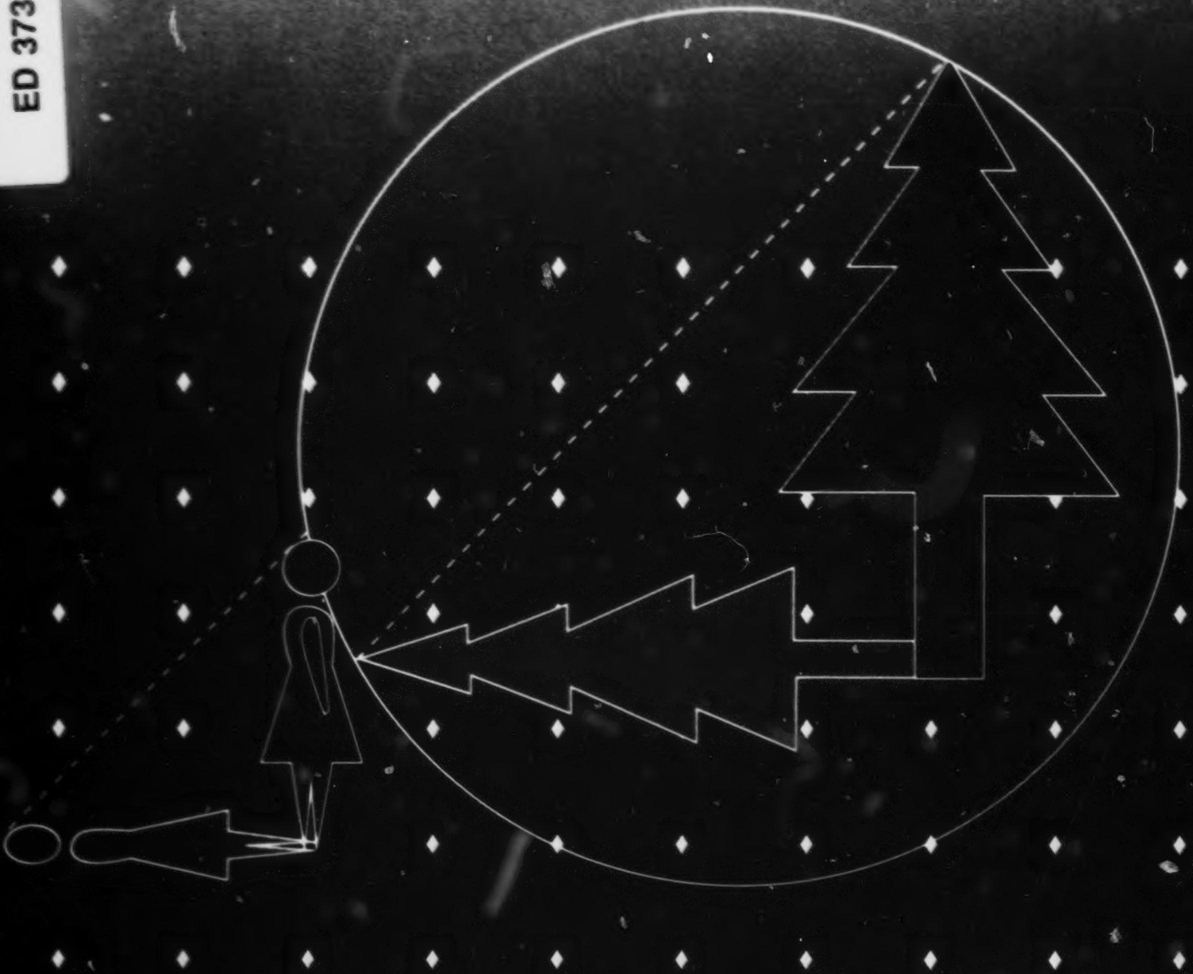
CURRICULUM AND EVALUATION

STANDARDS

FOR SCHOOL MATHEMATICS

MEASUREMENT IN THE MIDDLE GRADES

ED 373 990



GRADES 5–8

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

"PERMISSION TO REPRODUCE THIS MATERIAL IN OTHER THAN PAPER COPY HAS BEEN GRANTED BY

J.T. Carpenter

TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)"

U.S. DEPARTMENT OF EDUCATION
Office of Educational Research and Improvement
EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)

☒ This document has been reproduced as received from the person or organization originating it.

☐ Minor changes have been made to improve reproduction quality.

- Points of view or opinions stated in this document do not necessarily represent official OERI position or policy.

BEST COPY AVAILABLE

NCTM
MATHEMATICS:
EVERYBODY'S HERITAGE,
EVERYBODY'S FUTURE.
75th
ANNIVERSARY
NATIONAL
COUNCIL OF
TEACHERS OF
MATHEMATICS

NEW PRODUCT ANNOUNCEMENT

▼ ▼ ▼ ▼ ▼ ▼ ▼ ▼ ▼ ▼ ▼ ▼ ▼ ▼ ▼ ▼ ▼

For further information contact:
Tom Pearson, Advertising Assistant
(703) 620-9840 ext. 154

Reston, VA
August 1994

Title: ***MEASUREMENT IN THE MIDDLE GRADES: Addenda Series, Grades 5–8***

Author: Dorothy Geddes and others

List Price: $12. Stock #588. The book may be purchased directly from NCTM at the address below or by calling (800) 235-7566. A free catalog is available on request.

Other Information: Softcover, 8½" x 11", 87 pages, ISBN 0-87353-367-4. Copyright 1994.

About the Book: This new offering in the popular NCTM Addenda to the *Curriculum and Evaluation Standards for School Mathematics* series presents activities that focus on learning measurement as it is used in the real world. The investigative approach used in these activities is designed to encourage middle-school students to explore ways of approaching real-life problems, to discover patterns, and to make and test hypotheses. These activity-oriented explorations are geared to strengthen students' higher-order thinking skills and estimation.

The twenty-four activities in the book are clustered around three themes—estimating and measuring; discovering patterns and logical connections to create measurement formulas, and making connections through interdisciplinary measurement activities.

Each cluster of investigations includes teacher notes with objectives and materials, easily duplicated activity sheets, and suggestions for integrating technology and alternative assessment.

Audience: Mathematics teachers, supervisors, curriculum developers, and teacher educators in grades 5–8.

For a Review Copy: Mail the enclosed form to Tom Pearson at NCTM, Department P, 1906 Association Drive, Reston, VA 22091-1593 or fax to (703) 476-2970. For information call (703) 620-9840 ext. 154. **Please send review clippings to the address above.**

▲ ▲ ▲

BEST COPY AVAILABLE

CURRICULUM AND EVALUATION

STANDARDS

FOR SCHOOL MATHEMATICS

ADDENDA SERIES, GRADES 5–8

MEASUREMENT IN THE MIDDLE GRADES

Dorothy Geddes

with

Robert Berkman
Iris Fearon
Michael Fishenfeld
Caroline Forlano
David J. Fuys
Jodi Goldstein
Rosamond Welchman

Consultants

Diana V. Lambdin
Frank K. Lester, Jr.
Kenneth P. Goldberg

Frances R. Curcio, Series Editor

NATIONAL COUNCIL OF
TEACHERS OF MATHEMATICS

Copyright © 1994 by
THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS, INC.
1906 Association Drive, Reston, Virginia 22091-1593
All rights reserved

Library of Congress Cataloging-in-Publication Data

Geddes, Dorothy.
Measurement in the middle grades / Dorothy Geddes, with Robert Berkman ... [et al.].
p. cm. — (Curriculum and evaluation standards for school mathematics addenda series. Grades 5–8)
Includes bibliographical references.
ISBN 0-87353-367-4
1. Mensuration. 2. Mathematics—Study and teaching (Elementary) I. Title. II. Series.
QA465.G37 1993
530.8'071'2—dc20 93-45338
CIP

The publications of the National Council of Teachers of Mathematics present a variety of viewpoints. The views expressed or implied in this publication, unless otherwise noted, should not be interpreted as official positions of the Council.

Printed in the United States of America

TABLE OF CONTENTS

FOREWORD iv

PREFACE v

INTRODUCTION 1

Why Measure? 1
Purpose of This Book 2
Estimating and Measuring 2
Discovering Patterns and Logical Connections 3
Making Connections 5
Summary 6

SAMPLE ACTIVITIES 7

Cluster A: Estimating and Measuring 10
Cluster B: Discovering Patterns and Logical Connections to Create Measurement Formulas 29
Cluster C: Making Connections through Interdisciplinary Measurement Activities 52

BIBLIOGRAPHY 80

APPENDIX: BLACKLINE MASTERS 83

6

FOREWORD

In March 1989, the National Council of Teachers of Mathematics officially released the *Curriculum and Evaluation Standards for School Mathematics* (NCTM 1989). The document provides a vision and a framework for strengthening the mathematics curriculum in kindergarten to grade 12 in North American schools. Evaluation, an integral part of planning for and implementing instruction, is an important feature of the document. Also presented is a contrast between traditional rote methods of teaching, which have proved to be unsuccessful, and recommendations for improving instruction supported by current educational research.

As the *Curriculum and Evaluation Standards* was being developed, it became apparent that a plethora of examples would be needed to illustrate how the vision could be realistically implemented in the K–12 classroom. A Task Force on the Addenda to the *Curriculum and Evaluation Standards for School Mathematics*, chaired by Thomas Rowan and composed of Joan Duea, Christian Hirsch, Marie Jernigan, and Richard Lodholz, was appointed by Shirley Frye, then NCTM president. The Task Force's recommendations on the scope and nature of the supporting publications were submitted in the fall of 1988 to the Educational Materials Committee, which subsequently framed the Addenda Project for NCTM Board approval. Following the release of the *Curriculum and Evaluation Standards* and on the basis of recommendations of the NCTM Task Force (Rowan 1988), writing teams were assigned to develop addenda to provide teachers with classroom ideas for translating the *Standards* into classroom practice. Three writing teams were formed to prepare materials for grades K–6 (Miriam A. Leiva, editor), grades 5–8 (Frances R. Curcio, editor), and grades 9–12 (Christian R. Hirsch, editor).

The themes of problem solving, reasoning, communication, connections, technology, and evaluation have been woven throughout the materials. The writing teams included K–12 classroom teachers, supervisors, and university mathematics educators. All the materials have been field tested in an effort to make them "teacher friendly."

Implementing the *Curriculum and Evaluation Standards* will take time. The Addenda Series was written to help teachers as they undertake this task. Furthermore, the Addenda Series is appropriate for use in inservice staff development as well as for use in preservice courses in teacher education programs.

On behalf of the National Council of Teachers of Mathematics, I would like to thank all the authors, consultants, and editors who gave willingly of their time, effort, and expertise in developing these exemplary materials. In particular, I would like to acknowledge gratefully the work of Dorothy Geddes and her contributing authors, Robert Berkman, Iris Fearon, Michael Fishenfeld, Caroline Forlano, David Fuys, Jodi Goldstein, and Rosamond Welchman, for preparing *Measurement in the Middle Grades*. Special thanks are extended to Diana V. Lambdin and Frank K. Lester, Jr., evaluation consultants, and Kenneth P. Goldberg, technology consultant, for their contributions to the entire Grades 5–8 Addenda Series. Finally, this project would not have materialized without the outstanding technical support supplied by Cynthia Rosso and the NCTM staff.

Bonnie H. Litwiller, *Addenda Project Coordinator*

PREFACE

The purpose of *Measurement in the Middle Grades*, as well as of the other books in the Grades 5–8 Addenda Series, is to provide teachers with ideas and materials to support the implementation of the *Curriculum and Evaluation Standards for School Mathematics* (NCTM 1989). In addition to this book, the five other publications in this series are *Understanding Rational Numbers and Proportions* (Curcio and Bezuk forthcoming), *Patterns and Functions* (Phillips 1991), *Developing Number Sense in the Middle Grades* (Reys 1991), *Dealing with Data and Chance* (Zawojewski 1991), and *Geometry in the Middle Grades* (Geddes 1992). These books are *not* an outline of a middle school curriculum, but rather they present several topics and activities to exemplify the ideas advocated in the *Curriculum and Evaluation Standards*, and they provide examples to help students make the transition from elementary to high school mathematics.

The unifying themes of the *Curriculum and Evaluation Standards*, the characteristics of a new classroom learning environment, and the role of assessment are described below. The discussion in each topic will refer to examples in this book.

Unifying Themes

The unifying themes of the *Curriculum and Evaluation Standards* include mathematics as problem solving, mathematics as communication, mathematical reasoning, and mathematical connections. These themes are not separate, isolated entities, but rather they are all interrelated. The ideas, examples, illustrations, and activities presented in this book are designed to demonstrate the interrelationships by weaving these themes throughout the activities as well as by providing ideas for incorporating technology and evaluation techniques.

Mathematics as problem solving. Although problem solving has been a goal of mathematics instruction throughout the years, it became the focus of attention with the advent of NCTM's *An Agenda for Action* (NCTM 1980). The *Curriculum and Evaluation Standards* reaffirms the importance of problem solving in mathematics instruction.

The excitement of learning and applying mathematics is generated when problems develop within the context of a situation familiar to students. Allowing them to formulate problems as they naturally arise within the context of everyday experiences gives them the opportunity to put mathematics to work, observing its usefulness and its applicability (e.g., Activity 18*a*). However, "not all problems require a real-world setting. Indeed, middle school students often are intrigued by story settings or those arising from mathematics itself" (NCTM 1989, p. 77). For example, see Activities 10 and 13.

Mathematics as communication. Although the formal language of mathematics is concise, is high in concept density, and may be seemingly foreign, students should have the opportunity to bring meaning to mathematics on the basis of their prior knowledge and background experiences. Allowing them to talk about their experiences and how they relate to mathematics concepts, listen to each other as they share ideas, read mathematics in various formats (e.g., number sentences, graphs, charts), and write about mathematical situations affords students the opportunity to compare experiences, clarify their thinking, and develop an understanding of how the mathematics they study in school is related to the mathematics they experience in the "real world." This requires the integration of the four language arts—speaking, listening, reading, and writing—with the mathematics lesson (e.g., Activity 6).

Communicating in mathematics requires a common language and familiarity with modes of representing mathematical ideas. Depending on students' "comfort level," oral language, prose, manipulatives, pictures, diagrams, charts, graphs, or symbols can be used to communicate ideas (e.g., Activities 7*a–b*). They should be encouraged to translate from one mode to another.

Mathematics as reasoning. During the middle school years, students should have opportunities to develop and employ their abilities in logical and spatial reasoning (e.g., Activity 16) as well as in proportional and graphical reasoning (e.g., Activities 1, 13, and 23*a*). The development of a student's reasoning ability occurs over a period of time. We can observe extreme differences between students in grade 5 and students in grade 8. As a result, instructional approaches must reflect these differences.

Depending on students' readiness, exploratory activities, experiments, and projects may require them to give a descriptive account of what they observe, an informal argument based on empirical results, or a formal proof supporting a conjecture (Hirsch and Lappan 1989). Active learners should be constantly involved in questioning, examining, conjecturing, and experimenting (e.g., Activities 9*a–d*, 20, and 21).

Mathematical connections. Traditionally, mathematics has often been presented as an isolated set of rules to be memorized. The *Curriculum and Evaluation Standards* suggests that mathematics be presented as an integrated whole. Students should observe the interrelatedness among branches of mathematics: number theory, geometry, algebra, probability, and so on (e.g., Activity 11). Furthermore, students should become aware of how mathematics is related to other disciplines such as science, architecture, art, literature, music, social studies, business, and industrial technology (e.g., Activities 19, 23, and 24).

There is no guarantee that allowing students to explore, create, and experiment within the context of a problem-solving setting will lead them to discover connections between and among mathematics concepts. Teachers may need to guide students in discovering connections and to elicit these connections explicitly. Students who recognize connections within mathematics and with other disciplines can understand and appreciate the logical unity and the power of mathematics (Steen 1989).

A New Classroom Environment

Implementing the *Curriculum and Evaluation Standards* requires a new way of teaching. The traditional teacher roles of authority figure and information disseminator must change to learning facilitator and instructional decision maker.

Knowledge about students and how they learn mathematics can contribute to establishing a conducive learning environment for middle school students. The teacher selects the instructional objectives on the basis of knowledge of his or her students, knowledge of mathematics, and knowledge of pedagogy (NCTM 1991). After selecting the instructional objectives, the teacher must decide how to deliver the content. Is the use of manipulatives appropriate? Is the use of technology appropriate? Is a cooperative learning setting appropriate?

Appropriate use of manipulatives. Manipulatives are multisensory tools for learning that provide students with a means of communicating ideas by allowing them to model or represent their ideas concretely. Using manipulatives, however, does not guarantee the understanding of a mathematics concept (Baroody 1989). After allowing students to explore using manipulatives, teachers must formulate questions to elicit the important, "big" mathematical ideas that enable students to make connections between the mathematics and the manipulatives used to represent the concept(s) (e.g., Activities 10 and 11).

Appropriate use of technology. Developments in technology have made the traditional, computation-dominated mathematics curriculum obsolete. As a result, technology has been given a prominent place in the *Curriculum and Evaluation Standards*, in terms of which technology should be made available for use in the classroom and how technology should be used in mathematics instruction. It is expected that at the middle school level, students will have access to appropriate calculators, and computers should be available for demonstration purposes as well as for individual and group work (NCTM 1989, p. 8). It should be noted that new advances in technology are being made constantly. Teachers should keep abreast of new developments that support mathematics instruction.

In this book, suggestions are made for integrating the use of computers to organize and analyze data (e.g., Activities 6 and 23*a*), to study relationships between and among geometric shapes (e.g., Activity 13), and to test and examine conjectures (e.g., Activity 13). Specifically, the use of MECC Create-a-Base, Euclid's Toolbox, the Geometer's Sketchpad, and the Geometric Supposer are used in some activities.

Integrating the use of calculators is also recommended. In particular, calculators with square root and fraction capabilities are employed to facilitate and expedite computation (e.g., Activities 11, 12, and 16).

Appropriate use of cooperative learning groups. Traditionally, mathematics has been taught as a "solo," isolated activity, yet in business and industry, mathematicians often work in teams to solve problems and attain common objectives (Steen 1989). Allowing students to work in cooperative groups affords them the opportunity to develop social and communication skills while working with peers of various ethnic, religious, and racial groups.

Cooperative learning environments, characterized by students working together and interacting with each other, contribute to internalizing concepts by forcing students to defend their views against challenges brought by their peers. The value of this approach is supported by the work of Vygotsky ([1934]1986), who discussed the increasingly interrelated nature of language and cognition as children grow.

Cooperative groups usually contain three to five students and may be established for various lengths of time (Artzt and Newman 1990; Davidson 1989). Unlike most traditional small-group instruction in reading and mathematics, cooperative learning groups are heterogeneous, and everyone must work together for the common good of all. Students who understand the concept being discussed are responsible for explaining it to those who do not understand. Examples for using cooperative learning can be found in Activities 18*b* and 24*a*. When using cooperative groups, teachers must consider new ways of evaluating student performance to ensure the success of the instructional objective(s).

The Role of Assessment

Making changes in the content and methods of mathematics instruction will also require making changes in why and how students' work is assessed. Assessment should be an integral part of instruction, not limited to grading and testing. According to Kroll and Lester (1991), there are at least four reasons for collecting evaluation information:

- To make decisions about the content and methods of mathematics instruction
- To make decisions about classroom climate
- To help in communicating what is important
- To assign grades

In other words, assessment includes much more than marking right and wrong answers. It "must be more than testing; it must be a continuous, dynamic, and often informal process" (NCTM 1989, p. 203). What methods can be used for assessment purposes? The *Curriculum and Evaluation Standards* recommends that teachers use a variety of types of evaluation. Kroll and Lester (1991) discuss four major categories of evaluation: (1) *observing and questioning students*, (2) *using assessment data reported by students*, (3) *assessing students' written mathematics work*, and (4) *using multiple-choice or short-answer tests*. Teachers might, for example, observe and question students to assess their understanding and to gain insight into their feelings and their beliefs about mathematics; use holistic scoring techniques for a focused assessment of students' written problem-solving work; or collect information through students' responses to short-answer questionnaires or through written assignments such as journal entries or brief essays. The use of these multiple methods of collecting assessment data will contribute to a thorough evaluation of students' work. These and other evaluation techniques are discussed in more detail in Kroll and Lester (1991). This book focuses on the use of student journals and portfolios to assess students' work in each of the three clusters of activities.

This brief description of the unifying themes of the *Curriculum and Evaluation Standards*, the characteristics of a new learning environment, and the role of assessment is furnished as a starting point to understand and to appreciate the ideas that are presented in this book. It is hoped that these ideas will furnish a foundation for developing concepts in measurement in the middle grades and will generate interest among teachers for improving instructional and assessment techniques in mathematics.

Frances R. Curcio, Editor
Grades 5–8 Addenda Series

INTRODUCTION

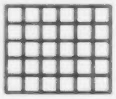
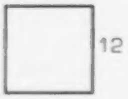
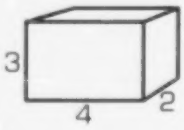
Why Measure?

Measurement is an intrinsic part of our everyday living. Whether we go shopping, visit a sports event or a bank, stay at home (doing carpentry, decorating, or working on hobbies), or even take a vacation (traveling and map reading), measurement is an integral part of our experiences. Our modern technology is wholly dependent on measurement.

Students need to perceive mathematics as a discipline of reasoning that enables them to solve real-world problems. Middle grades students encounter measurement ideas both in and out of school, and measurement experiences show them practical applications of mathematics. A dynamic interaction between students and their environment is required in measurement activities. Challenges, similar to the following, intrigue middle school youngsters and cause them to ask questions like these: What other data or information must we gather? What assumptions must we make? What strategies can we use to solve the problem?

> A class has decided on a paper-recycling project for its school. The students found that recyling companies pay different rates depending on the type and quality of paper they receive—newspaper, bond paper, or computer paper, for example. In order to plan for collecting, packing, and shipping the paper, they need to gather more data. How should they begin?

Measurement has been identified as one of the twelve components of essential mathematics for the twenty-first century by the National Council of Supervisors of Mathematics (1989), which noted that "students should learn the fundamental concepts of measurement through concrete experiences" (p. 472). Yet the poor responses of seventh graders to measurement questions reported in *Results from the Fourth Assessment of the National Assessment of Educational Progress* (Lindquist 1989) indicate students' lack of conceptualization of measurement ideas. Students had exceptional difficulty with problems involving perimeter, area, and volume. Sample questions and the percent of correct responses for seventh-grade students are shown below:

	% correct
1. What is the distance around a 4 by 7 rectangle? (Lindquist 1989, p. 40)	37
2. What is the area of this rectangle? (Lindquist 1989, p. 40) 	46
3. What is the area of this square? (Lindquist 1989, p. 164) 	13
4. What is the volume of this rectangular solid? (Lindquist 1989, p. 42) 	33

Since the concepts in these problems are a basic part of the middle grades mathematics curriculum, how can we account for the low success rate? Do we provide sufficient opportunities for the students to engage in hands-on real-world experiences designed to develop an understanding of basic measurement concepts?

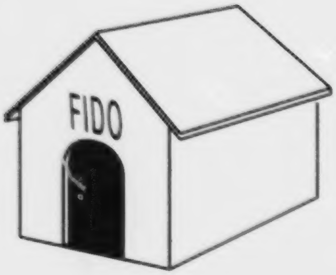


Fig. 1

Under the caption "Mathematics is a science of pattern and order" in *Everybody Counts*, the National Research Council (1989, p. 31) notes how the ideas of mathematics influence the way we live and work. In particular, they illustrate how a practical knowledge of mathematics and measurement can be put to immediate use to solve problems. For example:

> Design a doghouse that can be made from a single 1 m × 2 m sheet of plywood. Make the doghouse [see fig. 1] as large as possible and show how the pieces can be laid out on the plywood before cutting.

This type of problem challenges students to think mathematically—to find the best solution by asking "What if..." questions and exploring all possibilities. In *Counting on You*, the National Research Council states that "goals for student performance are shifting from a narrow form of routine skills to development of broad-based mathematical power" (1991, p. 5). This shift in focus will enhance students' ability to reason logically and to use a range of mathematical and measurement methods to solve a wide variety of nonroutine real-world problems.

So important is the study of measurement that the National Council of Teachers of Mathematics (NCTM) has (1) focused on the need for "extensive concrete experiences using measurement" (p. 116) in Standard 13 for grades 5–8 in the *Curriculum and Evaluation Standards for School Mathematics* (NCTM 1989), (2) addressed "Teaching Measurement Estimation" in a major section of the 1986 Yearbook (Coburn and Shulte 1986), (3) included a chapter entitled "Developing Measurement Sense" (Shaw and Cliatt 1989) in the 1989 Yearbook, and (4) highlighted major articles on measurement in recent issues of the *Arithmetic Teacher* (Fay and Tsairides 1989; Wilson and Adams 1992).

Purpose of This Book

This book addresses issues and concerns about teaching and learning measurement in the middle grades and provides sample activities with a focus on learning measurement *for a purpose*. The activities are clustered around three themes:

- Estimating and Measuring
- Discovering Patterns and Logical Connections to Create Measurement Formulas
- Making Connections through Interdisciplinary Measurement Activities

Problem solving is highlighted in each cluster. The investigative approach used in the activities is designed to encourage students to explore ways of approaching real-world problems, to discover patterns, and to make and test hypotheses. These activity-oriented explorations will nurture students' insights into the process of measurement and into the usefulness and power of mathematics in solving problems. Furthermore, students' estimation and higher-order thinking skills will be strengthened by these experiences.

"A great discovery solves a great problem but there is a grain of discovery in the solution of any problem." (Polya 1973, p. v)

Estimating and Measuring

If measurement is to be a useful tool, estimation becomes one of the important goals of teaching measurement. Estimation requires students to make a judgment of an object in relation to some standard. Extensive classroom experiences in estimation are needed so that students will develop estimation strategies (Bright 1976). Three common estimation strategies helpful to students include (1) having a model or referent (e.g., a doorknob is approximately one meter from the floor); (2) partitioning an object to be estimated into parts for which students know the measure (sometimes referred to as "chunking"); and (3) dividing up the object into a number of equal parts, called "unitizing" (Osborne 1980; Lindquist 1987).

To improve estimation skills, frequent estimation practice is required; thus the theme should be "*estimate and then measure*" (Coburn and Shulte 1986). Students should aim to make the difference between the estimate and the measure as small as possible. They might compare their estimates and measurements of given objects by making a graph (see fig. 2). This will provide a visual image that shows whether they have a tendency to overestimate or underestimate. An enjoyable way for students to improve their estimation skills is to play a game (see Activity 2) scored by finding the differences between estimates and the actual measurements of objects, the winning score being the smallest sum of error differences.

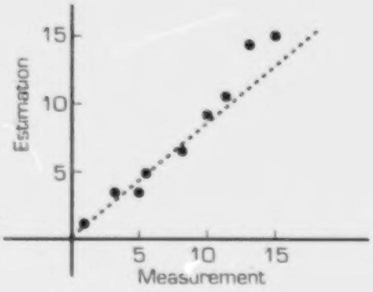


Fig. 2

"The teacher of mathematics should orchestrate discourse by posing questions and tasks that elicit, engage, and challenge each student's thinking." (Professional Standards for Teaching Mathematics [NCTM 1991, p. 35])

Of course there are times in real-world situations when a good estimate rather than a precise measure is all that is needed. For example:

> A friend wanted to find the cost (at \$10 per m^2) of carpeting a rectangular area 7.5 m by 3.5 m. She decided to estimate the cost by changing the dimensions to 7 m by 4 m, and then to check, she used the dimensions 8 m by 3 m. She thought she justified her changes by keeping the perimeters the same—dropping 0.5 m from one side and adding 0.5 m to the other side each time. However, she did not get the same answer! Did she get a good estimate? (*Note:* Imbedded in this problem is a further investigation question—namely, do two shapes with equal perimeters have equal areas?)

Many problem-solving opportunities should be provided for students to decide when an exact answer is required and when a good estimate will serve the purpose. As students recognize estimation as a practical skill, they will see the need to become good estimators. The teacher's role is vital in creating classroom settings that foster students' hands-on measurement activities. When a teacher sets up cooperative learning groups or learning centers in the classroom, students have the opportunity to communicate about various measurements and to discuss the need for, and importance of, estimating and using standard units.

In implementing measurement activities, teachers need to provide a variety of measuring tools. Students have to learn how to select an appropriate tool and unit for a measurement task depending on the degree of accuracy required. Experience in making some of these tools gives students valuable insights into their use. To foster communication among students and to assess their understanding of measuring units after they have completed many activities, problems similar to those in figure 3 might be considered. Students are asked to circle the choice that best completes the statement and then to write one or two sentences about each of the other choices to explain why they are not appropriate. Samples of different measuring units should be readily available in the classroom for reference. Students in cooperative groups should then share and discuss their responses, correct and explain any misconceptions, and identify those situations where they would like to have more experience.

An average person can—

a. plant a tulip bulb in one cubic centimeter of dirt;

b. run one kilometer in a minute;

c. make ice cubes in a freezer set at 10 degrees Celsius;

d. cover the top of a card table with one square meter of cloth;

e. buy a meter of milk at a grocery store;

f. find a pencil that is one milligram long.

Fig. 3

Discovering Patterns and Logical Connections

Measurement experience is a powerful mathematical connection among topics in the middle grades mathematics curriculum. For example, most models that are used to explain numerical operations with rational numbers are based on measurement concepts. Also congruence, similarity, and other aspects of geometry are interconnected with measurement, as seen in the following example:

> Students were asked to look for applications of mathematics outside their school. Pat, who is 1.5 meters tall, was walking along a sidewalk when her friend measured Pat's shadow and found it to be 1 meter long. They measured the shadow of a nearby tree and found it was 3 meters. They asked, "How can we use math to find the height of the tree without climbing the tree?" (See fig. 4.)

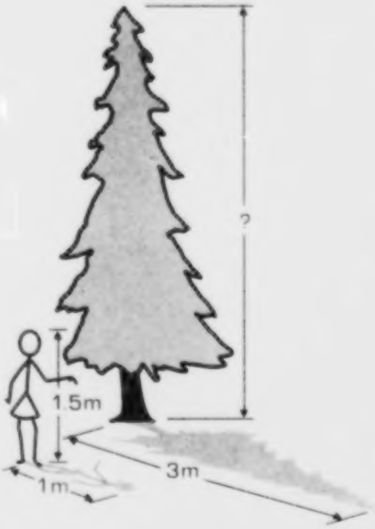


Fig. 4

Initial student experiences with a measurement system involving a particular attribute (e.g., length, area, volume, capacity, weight (mass), time, numerosity, rate, density) should include making number-free comparisons and then using nonstandard units of measure (e.g., handspans, strides, index cards, jelly beans). Following these experiences, it is necessary to focus on the fundamental characteristics of any measurement system, namely, the need to assign a number, to make comparisons, to recognize and use the concept of congruence, to select a unit, and to realize that, when appropriate, congruent units can be added (see Wilson and Osborne [1992]).

As middle grade students investigate problems involving measurement and develop their own procedures for finding measures of one-, two-, and three-dimensional figures, they will make generalizations and deductions—and thus create formulas. Too often it has been found that students merely memorize formulas and plug in numbers without understanding what the formulas mean or how they are related to one another. To develop students' insight into measurement units, engage them in hands-on experiences—using string for measuring length (i.e., perimeter), using square units (tiles) for *covering* a region (i.e., area), and cubic units (cubes) for *filling* a space (i.e., volume). After students have had considerable experience in finding the perimeter, area, and volume of figures, they might be challenged by the following investigation: Compare the surface areas and volumes of several different sizes of milk cartons; make a table and record the figure, dimensions, surface area, and volume. What do you notice? What conjectures would you make? (See Activities 14 and 15.) The activities should always be designed to encourage students to search for patterns (i.e., *inductive reasoning*, as in the milk-carton investigation). For a further example of inductive reasoning, see Activity 10, showing the development of Pick's theorem for areas of geoboard figures.

"Mathematics presented in the Euclidean way appears as a systematic, deductive science; but mathematics in the making appears as an experimental, inductive science." (Polya 1973, p. vii)

Students should also have opportunities to engage in *deductive reasoning*, as shown in the following illustration for deducing the formula for the area of a parallelogram if we know that the area of a rectangle is equal to the product of the base and the height drawn to that base.

> Each student is given a paper rectangle and a paper parallelogram (with no right angle) whose bases have the same measure and whose heights have the same measure. (See fig. 5.) What properties are the same and what properties are different in the two shapes? Explain. Now cut the parallelogram into two pieces (by cutting along the indicated height) and reassemble the two pieces in a different way so that two sides match. What discoveries have you made? How can you justify or deduce that a newly assembled figure is a rectangle and that it is congruent to the given rectangle? What can you then deduce about the area of the given parallelogram? Of any parallelogram? Explain.

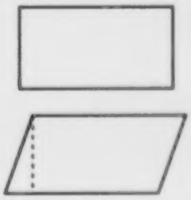


Fig. 5

Thus students can see how the area of a parallelogram can be *deduced* from the area of a rectangle. Students can also be challenged to discover other interrelationships among area formulas and to build "family trees" to show alternative ways of finding areas of figures (see Activity 9*d*). For example, if we know the area of a square, how can we find the area of a circle? Or knowing the area of a rectangle and a circle, how

BEST COPY AVAILABLE

can we find the surface area of a right circular cylinder? In a similar way, through hands-on experiences, students can investigate and generate new discoveries and generalizations for finding volumes of three-dimensional figures in real-world settings (see Activity 15).

Making Connections—Interdisciplinary Measurement Activities

In our efforts to quantify real-world situations in other disciplines (e.g., science, social studies, art), many ideas about number and measurement also arise. Observations and real measurement from science experiments can be used effectively in mathematics lessons. Especially in this context, the approximate nature of measure is an aspect of number that needs particular attention in the middle grades. Because of students' prior experience with counting and operations with numbers that yield exact answers, it is sometimes difficult for them to develop the concept of the approximate nature of measuring. Only after considerable experience do students recognize that when they correctly measure to the nearest "unit," the maximum possible error would be one-half that unit (e.g., if measuring to the nearest centimeter, the error could be 0.5 cm; if measuring to the nearest meter, the error could be 0.5 m). Students should not think of the error of a measurement as a mistake but recognize that it is a result of the limitations of the measuring device being used. While engaged in observation and measurement processes, students must discover and discuss how certain acts (e.g., selection and use of measurement tools) affect the precision and accuracy of their measurements.

Many patterns and relationships are discovered by students as they engage in investigations involving measurement. These investigations should extend beyond the mathematics classroom to include other disciplines (e.g., science, social sciences, mathematics, health and physical education). For example: A seventh-grade sports class is engaged in a mini-Olympics standing long jump and gathered the data shown in figure 6. Questions such as the following should be considered: Are there any observable patterns? Is jump length related to height? To sex? Are more data needed? What if we graph the data? Would we get the same results in a fifth-grade class? In an eighth-grade class?

Height (in cm)	Sex	Jump length (in cm)
110	F	100
160	F	120
151	M	123
126	M	135
100	F	91
125	M	121
141	F	108
162	M	128
132	F	102
143	M	115

Fig. 6

If a science class is to determine the effect of temperature on the expansion of a steel bar, the students need to explore how various temperature states of the bar correspond to changes in the length of the bar. Similarly in social studies classes, if databases are used or created, what relationships among the data can be discovered (see Bright [1989] and Budin, Kendall, and Lengel [1986])? These experiences provide students with many opportunities to gather data, make comparisons, look for patterns, and formulate conjectures. By furnishing students with challenging interdisciplinary measurement activities, teachers promote open inquiry.

Middle school activities should also be designed to develop an understanding of the concept of rate. Within a classroom setting and with the use of a stopwatch, students can experience and see different rates. For example: Fasten a 10-meter length of tape to the floor and have students, walking as rapidly as they can, traverse the tape—first using "baby" steps, then normal steps, and finally "giant" steps—with the distance, time in seconds, and rate recorded for each trip as shown in figure 7. Extensions of this idea of finding rates in sports activities and travel (where students are personally involved) help them gain insight into the notion that rate is distance measured in relation to units of time. Similar types of activities can help students develop an understanding of the concept of density (see Activity 24). Personal experiences of this type help students recognize and appreciate the use of measurement concepts in other real-world settings.

	Distance	Time	Rate
"Baby" steps	10 m	30 s	0.33 m/s
Normal steps	10 m	20 s	0.5 m/s
"Giant" steps	10 m	10 s	1 m/s

Fig. 7

Measurement activities such as constructing scale drawings and scale models of a room, garden, or local community (see Activity 18) or relating biological growth and form (see Activity 23) provide excellent opportunities for students to engage in proportional reasoning. Activities on field trips that enable students to use a variety of measuring instruments (some student-made) to find measures of inaccessible objects by employing indirect measurement techniques and proportional reasoning (see Activity 14) generate much enthusiasm.

Summary

This sampling of issues and ideas related to measurement provides evidence that measurement is an integral and important part of everyday living. Throughout the study of the middle grades mathematics curriculum students can be involved in measurement activities that help them see mathematics as a dynamic area of inquiry. A case has been made for establishing a classroom atmosphere that furnishes opportunities for students to engage in activity-oriented investigations of real-world problems that not only nurture reasoning and problem-solving skills but also allow students to perceive first hand the power and practicality of mathematics. A few such explorations have been highlighted in the discussion above. A more comprehensive set of such activities with accompanying teacher notes is presented in the next section, entitled "Sample Activities." The schema (fig. 8) indicates the organization and sequence of the Sample Activities. Most activities involve student investigations leading to discoveries, pattern finding, making and testing conjectures, and problem solving.

- Estimating and Measuring
  - How Good Is Your Estimate?
  - Estimate and Measure
  - Estimation Fair
  - Measuring Me
  - How Big Is Your Foot?
  - Average ***N***th Grader
  - Metric Olympics
  - Metric Models
- Discovering Patterns and Logical Connections to Create Formulas
  - PeRIMeter and Areas
  - Geoboards and Pick's Theorem
  - Squares and the Pythagorean Theorem
  - Measuring a Circle
  - Measuring Similar Figures
  - Surface Area
  - Volume of Polyhedra and Spheres
- Making Connections through Interdisciplinary Measurement Activities
  - Planet Jumping
  - Paintbrush Measure
  - Scale Models
  - Field Mathematics
  - Effect of Change in Size
  - Effect of Change in Scale
  - Size and Form of Buildings
  - Size and Form of Animals
  - Density

Fig. 8

SAMPLE ACTIVITIES

In this section, there is a variety of activities appropriate for the middle grades with a focus on learning measurement *for a purpose*. Each cluster of activities contains Teaching Notes and references to other sources for additional activities. The activities are designed around measurement topics in the mathematics curricula for grades 5–8 and incorporate many of the features recommended in the NCTM *Curriculum and Evaluation Standards*—namely, student investigations, activity-oriented explorations (constructions and the use of manipulatives), student writing, cooperative learning, spatial visualization activities, use of technology, making mathematical connections, van Hiele type of approaches to developing thinking and reasoning skills, problem solving with "What if..." extensions, and real-world applications.

Topics

As a prerequisite for the activities in this book, students should have some basic knowledge of metric units of measure—length, area, capacity (volume), weight (mass). The activities, organized in three clusters, are designed to enhance this knowledge and engage students in thinking and talking "metric" by providing hands-on measuring experiences in real-world settings.

Cluster A: Estimating and Measuring
Cluster B: Discovering Patterns and Logical Connections to Create Measurement Formulas
Cluster C: Making Connections—Interdisciplinary Measurement Activities

Activities in Cluster A engage the students in hands-on cooperative measurement projects. Cluster B provides the students with opportunities to discover generalizations (formulas) for finding perimeter, area, surface area, and volume. Activities involving measurement concepts in enrichment and interdisciplinary settings are given in Cluster C. Many of the activities within each cluster are self-contained. Motivational hands-on activities, problems, real-world applications, or puzzle situations are furnished to encourage student explorations. Most activities involve student investigations leading to discoveries, pattern finding, making and testing conjectures, and problem solving.

Focus on Metric Units of Measure

Throughout this book, metric units are used for measurement. Students should be encouraged to think in terms of metric units by associating the units with meaningful models—for example, with units on their own metric ID card (see Activity 4) and with everyday observations (e.g., a doorknob is approximately one meter from the floor, buying a liter of soda). Students should not be asked to convert from metric to common English units or vice versa, but they should do meaningful conversions within the metric system. The hands-on activities, cooperative learning, and journal "writes" included in each cluster provide contexts for students to "talk metric."

Objectives and Materials

For each cluster of activities, objectives are stated. The focus is on *"estimate and then measure"* and problem solving. Some activities address multiple objectives. Many of the activities involve hands-on experiences using inexpensive teacher-made or student-made materials. The hands-on activities provide valuable experiences in estimation and

◆ ◆ ◆ ◆ ◆ ◆ ◆ ◆

measurement for students. The materials needed for each activity are listed. Each cluster of activities has blackline masters in the Appendix numbered to correspond to the appropriate activity.

Activity Sheets

An Activity Sheet is furnished for each activity. Two different formats are used. Many are designed to be given to students to work on in small cooperative groups or independently, and others might be done more appropriately by the teacher and class working together; if used in this way, the Activity Sheet might be considered a script for the teacher—this is especially true if the class has limited reading skills. Some Activity Sheets may require several lessons to complete, or one might choose to complete only portions of an Activity Sheet with a class. The Challenge Problems on the Activity Sheets are nonroutine and require higher levels of thinking. Solutions to selected challenge problems are provided in the Teaching Notes.

Teaching Notes

Each group of activities contains a brief set of Teaching Notes to give some guidance to the teacher on the design and intended use of the activities. Teaching ideas are presented within the notes as a guide to the development of the activities. Answers to key questions on the Activity Sheets used by the students in their explorations and investigations are provided in the Teaching Notes. Copies of the Activity Sheets should be duplicated for the students.

Technological Tools

Calculators (including the Explorer calculator) should be readily available to students for individual and group work. The activities involving computer technology make use of Euclid's Toolbox, the Geometric Supposer, the Geometer's Sketchpad, and databases. Other software focusing on informal geometry and measurement is commercially available but has not been included in these activities except as references.

Assessment

As stated in the Preface, new approaches to assessment need to be used. Assessment is an ongoing task. Teachers can learn a lot about students' progress in measurement by being observant during class activities. A sample checklist of things to be aware of, and to find evidence for, includes—

- improvement in students' thinking skills;
- improvement in students' estimation and measurement skills;
- improvement in students' explanations and ability to justify conclusions;
- improvement in students' ability to select and use suitable technology when such use is appropriate;
- increase in students' self-confidence;
- increase in students' willingness to explore ideas, make and test conjectures, take risks, and engage in divergent thinking;
- increase in students' repertoire of problem-solving strategies;
- increase in students' dynamic interaction with their environment;
- increased student awareness of real-world measurement applications;
- increased student willingness to consider alternative solutions to problems;
- increased ability of students to recognize connections of measurement to other aspects of mathematics and to other disciplines.

◆ ◆ ◆ ◆ ◆ ◆ ◆ ◆

Even though a wide spectrum of assessment techniques should be used (e.g., observation, writing, tests, problem exploration,...), this book focuses on the use of student *journals* and *portfolios* to assess students' work. Students assemble in their portfolios the best products of their work: their journal writing; their growing glossary; their models and measurement tools; their investigations, discoveries, conjectures, and verifications; their analyses of specific problems and possible extensions; their creative activities; their computer explorations; their projects. Illustrative examples of assessment activities are given in each cluster.

◆ ◆ ◆ ◆ ◆ ◆ ◆ ◆

CLUSTER A
ESTIMATING AND MEASURING

Activity 1:	How Good Is Your Estimate?
Activity 2:	Estimate and Measure Game
Activity 3:	Estimation Fair
Activity 4:	Measuring Me
Activity 5:	How Big Is Your Foot?
Activity 6:	Average *N* th Grader (*N* = 5, 6, 7, 8)
Activity 7:	Metric Learning Centers *a.* Length *b.* Metric Olympics *c.* Capacity *d.* Weight
Activity 8:	Metric Models and Projects

OBJECTIVES OF CLUSTER A:

1. To learn the concept of metric measurement by doing—through an active hands-on approach
2. To have students think metric—by relating metric measurements to themselves (e.g., body measurements) and to their home and community
3. To provide experiences with estimation and direct physical measurement of length, area, and capacity (volume) in cooperative learning settings
4. To provide experience in learning center contexts both as participants and as guides for activities
5. To understand the metric system as a system of measurement by recognizing relationships among the units and by doing conversions within the system
6. To have students construct and use selected measurement devices
7. To have students engage in data analysis by gathering, organizing, analyzing, interpreting, and reporting on data
8. To apply measurement concepts to real-world problem-solving situations

> ***"Problem solving is the process by which students experience the power and usefulness of mathematics in the world around them." (Curriculum and Evaluation Standards [NCTM 1989, p. 75])***

TEACHING NOTES FOR CLUSTER A

For middle grade students to solve measurement problems, basic measurement concepts must be known. The activities in Cluster A give students opportunities to develop an intuitive understanding of these concepts and to apply them to real-world measurement problem-solving situations. The focus of each activity is on *estimate and then measure.*

A brief historical background on the origin and development of the metric system might be given when the meter is reviewed as the standard unit of length. It should be noted that the metric system is a decimal system; decimeters, centimeters, and millimeters should be illustrated. Comparison of the metric units to the monetary system of the United States is useful.

◆ ◆ ◆ ◆ ◆ ◆ ◆ ◆

Below are listed specific Teaching Notes for each Activity Sheet including an indication of materials needed, teaching ideas, and answers to key questions.

Activity 1. How Good Is Your Estimate?

Materials: Copy of Activity Sheet 1 (two pages) for each student. Each student should have a 20-cm ruler (see Appendix), and metersticks should be available for group work.

This activity can be done in class and finished as a homework assignment that the students will put in their portfolio when completed. Class discussions of aspects of this assignment tend to be very lively. Students are interested in the interpretation of their graphs and need to be asked what an underestimate or overestimate means graphically. In general, you are overestimating if your dot is above the diagonal line and underestimating if your dot is below the diagonal line.

Assessment—journal entry: Write a paragraph in your journal explaining how you interpret your graph of measurements vs. estimates in terms of your estimation skills. Include a copy of your Activity Sheet with the graph.

Each student should make a folding meterstick (see Appendix), a mini-trundle wheel (see Appendix), a liter box, and a milliliter spoon (see Appendix) for use in class and at home.

Assessment—portfolio: Select some of the models you have constructed and include them in your portfolio; describe the models and how to use them.

Activity 2. Estimate and Measure Game

Materials: Copy of Activity Sheet 2 for each student; 20-cm ruler showing millimeters (see Appendix); Grab Bag with assortment of objects (e.g., straw, button, piece of string, toothpick, rubber band, bottle cap, soda can, paper clip, ballpoint pen, index card)

The Estimate and Measure Game can be played by a teacher against the class, by teams of students, or by students in small groups. A good way to start is for you to play a few rounds with them. Record results on a large score sheet on the chalkboard. You might select a few unusual measurements to spur students to do likewise—for example, "around an index card in centimeters" or "the length of a stretched rubber band in millimeters." Students enjoy thinking up unusual measurements to challenge their classmates. Be sure to have students report and record results using good metric language (e.g., saying and writing "12 cm" rather than just "12").

The game can be adapted for the estimation of capacity by using a liter container calibrated in milliliters and a collection of small containers (e.g., soup or juice cans, spoons, eye dropper). It can also be adapted for weight using gram weights on a primary balance scale and objects weighing from 10 g to 500 g.

These estimate-and-measure games can lead to follow-up activities that give further practice in measurement and that also stimulate mathematical thinking. Students might be challenged to select two objects whose combined lengths come closest to a certain length, say 25 cm. Or have the students use clues to identify mystery objects—for example, "28 cm around" (milk carton) or "the length is about 45 mm longer than the width" (small index card). Students should be encouraged to make up their own problems to challenge their classmates.

◆ ◆ ◆ ◆ ◆ ◆ ◆ ◆

All too often measurement activities deal only with measurement and neglect estimation or related problem solving. This Estimate and Measure Game builds an understanding of metric units, connects measurement with real-world objects, and leads to activities that promote problem solving and foster communication about ways to solve problems.

Answer to Challenge Problem 2: Suppose a student weighs 50 kg. The value of her weight in nickels is $500. (A nickel weighs 5 g.)

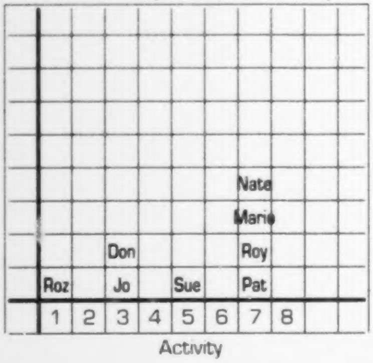
Activity 3: Estimation Fair

Materials: Copy of Activity Sheet 3 for each student, notebook (to write on), pencils. The individual activities require the following materials:

1. Liquid Measurement: Four different containers marked I, II, III, and IV; liter box (cubic decimeter), milliliter measuring cup (graduated cylinder), colored water
2. Weight: Various objects (e.g.,walnut, marble, sinker, block of wood, packet of pins, small bag of noodles), primary balance scale to weigh objects in grams, weights
3. Area: Transparent centimeter grid, square coaster, cut-out cardboard square decimeters, large polygon drawn on the ground with chalk
4. Body Measurement: Centimeter tape measure and ruler
5. Sponge Throw: Sponge, bucket of water. Mark a spot on the ground where students are to stand when they throw the wet sponge. (The wet sponge is fun to throw and leaves a temporary mark on the ground where it lands.)
6. From Here to There: A trundle wheel. Two points about 25 m apart should be marked A and B.
7. In a Minute: Jump rope, ball, minute timer
8. Volume: Two boxes (one that can easily be filled with cubic centimeters and one that cubic decimeters fit into nicely), cubic centimeters, cubic decimeters (made from six square cardboard decimeters)
9. Graphs: Two graphs prepared as described below.

At the start of the fair, give a brief orientation to explain procedures. When the students have finished all the activities, have two graphs (see fig. 9) available for students to enter their names. One graph should be labeled "What was your favorite activity?" and the other "In which activity was your estimate closest to the actual measurement?" Students should be told to write their names in a box over one activity, starting from the bottom of the column. In later class discussions, students might discuss what they mean by "closest." Perhaps least error as a percentage of the measurement? Or perhaps the least number of units? Students may enjoy designing their own learning stations for another Estimation Fair.

A. What was your favorite activity?



B. In which activity was your estimate closest to the actual measurement?

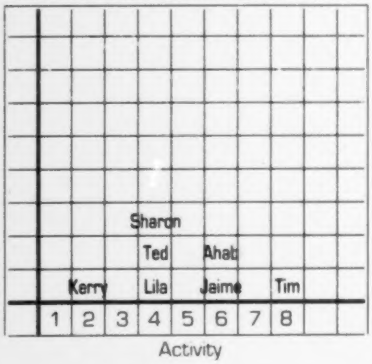


Fig. 9

Activity 4: Measuring Me

Materials: Copy of Activity Sheet 4 for each student

This sheet can be used for assessing students' understanding of the basic concepts of the metric system. The students might design their own metric ID card (perhaps include a photo) or design one that might be used for students in the lower grades in your school district. For other activities, see Coburn and Shulte (1986).

Activity 5: How Big Is Your Foot?

Materials: Copy of Activity Sheet 5 for each student; centimeter grid paper

Students sometimes have difficulties counting up regions that are not complete square units; they should be encouraged to estimate and to put together regions that approximate one square unit. Conjectures concerning relationships between their foot and the baby foot should be encouraged and explored.

Answer to the Challenge Problem: The part of the square covered by the circle is approximately 3/4.

Activity 6: Average Nth Grader (N = 5, 6, 7, 8)—Data Analysis

Materials: Copy of Activity Sheet 6 for each student

Some class discussion of the meaning of "average" is useful; also students should have some knowledge of basic statistical concepts (i.e., mean, median, mode). As a class or group project, students should design a survey form (similar to Activity Sheet 6) that will be given to all students on their grade level. The survey form will then be collected and analyzed to give a profile of the average *N*th grader. This activity is a perfect place to show students how to enter or manipulate data to answer given questions, using a database. For example, if each student enters her or his data from this activity into the database, they could find all the individuals who like the same favorite food, the number of different TV shows that are given as favorites, or the median amount of time spent watching television and doing homework each day. They can, using this information, create a make-believe student having all the "average" characteristics of the class as a whole. The class or group should then design a large display (for a school bulletin board or a district math fair) illustrating by graphs and charts the characteristics of an average *N*th grader. See fig. 10 for a photograph of such a display prepared by a seventh-grade class.

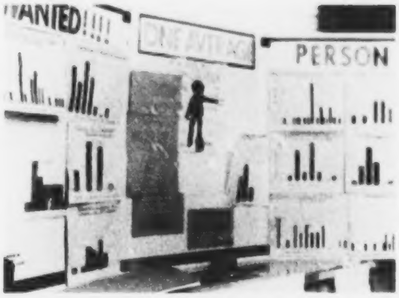


Fig. 10. Results from a project: "Wanted! An Average Seventh Grader"

The data from such surveys can also be used to enter as a database in a computer. Software such as Appleworks or MECC Create-a-Base is appropriate. Students can prepare spreadsheets on different aspects of the data they wish to analyze. If the survey is done at the beginning of the school year and again at the close of the school year, students can make analyses and comparisons to show changes that have occurred individually and collectively in the course of one school year. Students are enthusiastic about setting up and analyzing such databases and are usually quite fascinated with the results of these investigations.

Another example of data analysis is given in Activity 23*a*. For further ideas on data analysis projects, see Friel, Mokros, and Russell (1992).

Assessment—portfolio: Prepare a report describing your group's creation of a database, your use of a spreadsheet, and your analysis of selected data collected in your survey with accompanying charts, graphs, and other displays. Indicate how your project could be extended by asking some "What if..." questions, such as "What if we compared selected parts of these data (for example: class A with class B, homework time with TV time)? How would the results differ?"

Activity 7a–d: Metric Learning Centers

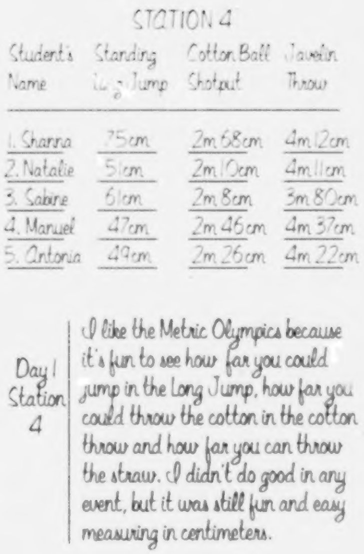
Materials: See cards for individual learning centers.

To prepare the learning center task cards for classroom use: Each page of learning center activities (pages 24 to 28) needs to be cut apart into individual tasks, and then each task should be enlarged to approximately 18 cm × 25 cm so it can be glued onto one-half of a letter-sized manila folder. A team of students can help prepare the cards by gluing each enlargement to one-half of a folder, adding a touch of color with magic

◆ ◆ ◆ ◆ ◆ ◆ ◆ ◆

markers to make a border or to highlight an item, and then laminating them or covering them with clear contact paper (if possible) for durability.

These learning center activities can be set up around the classroom for active hands-on experiences. Simple basic equipment, described on each card, is needed for each center. Students are most enthusiastic about these learning center tasks and actively engage in them. The data gathered from these activities should be analyzed in group and class discussions to determine the characteristics of the class—noting mean, median, mode, similarities, differences, patterns, relationships (e.g., is length of a jump related to height?), raising "What if ... " questions (e.g., if we recorded these data for students in another grade, how would their graphs compare with ours?), or writing about their "Olympics" experiences. See figure 11 for an illustration of how individual groups recorded their data and commentary.



STATION 4

Student's Name	Standing Long Jump	Cotton Ball Shotput	Javelin Throw
1. Sharna	75cm	2m 68cm	4m 12cm
2. Natalie	51cm	2m 10cm	4m 11cm
3. Sabine	61cm	2m 8cm	3m 80cm
4. Manuel	47cm	2m 46cm	4m 37cm
5. Antonia	49cm	2m 26cm	4m 22cm

Day 1
Station
4

I like the Metric Olympics because it's fun to see how far you could jump in the Long Jump, how far you could throw the cotton in the cotton throw and how far you can throw the straw. I didn't do good in any event, but it was still fun and easy measuring in centimeters.

Fig. 11

Assessment—journal entry: Write about your own brief experience and your team's experience in Metric Olympics.

Assessment—portfolio: Describe a new Olympic Metric Learning Center activity that your group constructed and place the Activity Card your group designed in your portfolio.

Activity 8: Metric Models and Projects

Materials: Copy of Activity Sheet 8 for each student.

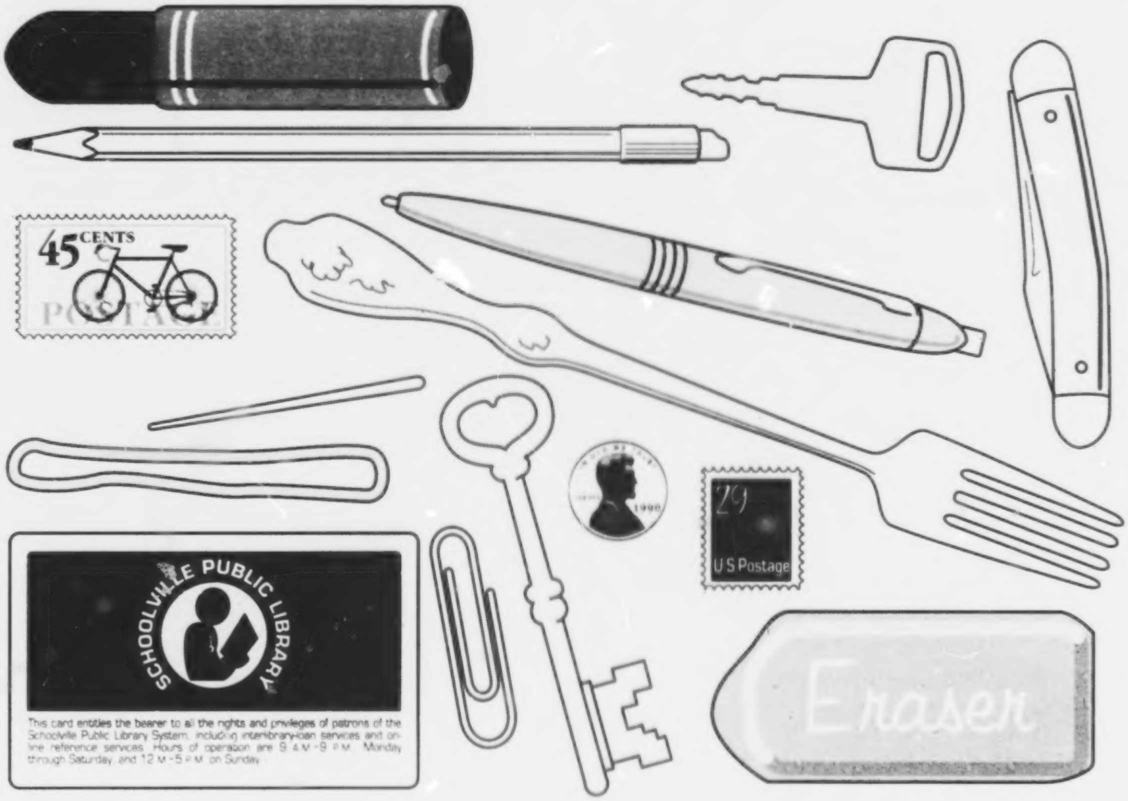
Assessment: This activity can be used for assessing students' understanding of the basic concepts of the metric system and their ability to apply this understanding in real-world problem-solving settings. Students should be asked to select two or three Challenge Problems from Cluster A and one Group Project and describe their approaches and strategies for exploring the problems and project, detail any difficulties encountered, and give partial, complete, and alternative solutions. They should also discuss possible ways of extending the problems and project.

Answers for the Challenge Problems: The use of calculators should be encouraged.

1. Have students think of and sketch different ways of placing cartons on the bottom of the storage space. Volume of each carton = 0.03 m^3. Volume of storage space = 7.5 m^3. There is ample room for 100 cartons.
2. Suppose one newspaper weighs 200 g. In one year each student will collect 260 kg of paper and earn $10.40. The class of 25 students would earn $260.
3. The Explorer (fraction) calculator's division key (INT ÷) could be used. Dividing 500 by 12 using this key gives an answer of 41 and a remainder of 8. Since they must buy whole numbers of liters, they would need 42 liters to make the trip. You might also elicit that the remainder of 8 means 8 out of 12 kilometers, and that represents 8/12, or 2/3, of that final liter they had to buy.

ACTIVITY 1
HOW GOOD IS YOUR ESTIMATE?

Estimation is an important aspect of measurement. For this activity, use the pictures of some common objects shown below.


45 CENTS
POSTAGE
29
US Postage
SCHOOLVILLE PUBLIC LIBRARY
This card entitles the bearer to all the rights and privileges of patrons of the Schoolville Public Library System, including interlibrary-loan services and on-line reference services. Hours of operation are 9 A.M.–9 P.M. Monday through Saturday, and 12 M.–5 P.M. on Sunday.
Eraser

First, for each object listed in the chart at the right, *estimate* the described length in centimeters, and enter your estimate in the appropriate column.

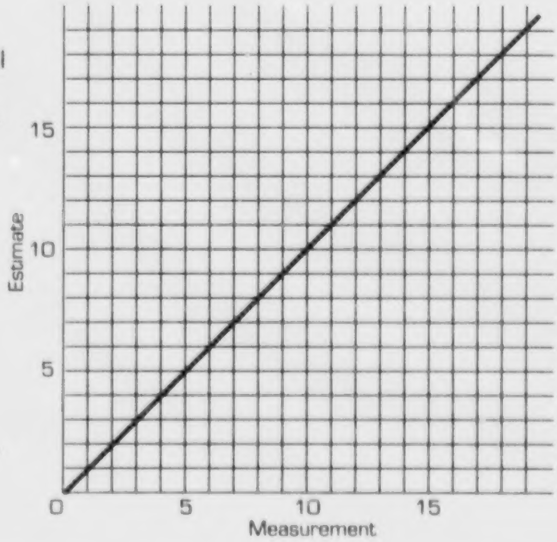
Then, when you have estimated *all* the lengths, use your ruler to measure them and enter the measurement of each object in the right-hand column.

Do you think that you estimate lengths in centimeters well? You can "see" how well you estimate by graphing your results on the graph paper on the next page.

Object	Estimate	Measure
Length of ballpoint pen		
Length of toothpick		
Width of paper clip		
Length of lipstick case		
Length of library card		
Length of pencil		
Perimeter of library card		
Length of bigger stamp		
Length of pocket knife		
Width of eraser		
Length of rubber band (if cut and opened up)		
Length of fork		
Length of longest key		

ACTIVITY 1 (CONTINUED)
HOW GOOD IS YOUR ESTIMATE?

Notice that on this piece of graph paper the horizontal axis is marked "Measurement" and the vertical axis is marked "Estimate." This means that if, for example, the actual measurement was 6 cm and you estimated 5 cm, you would start at the lower-left corner, go over 6 spaces to the right and up 5 spaces; put a dot at this point.



Do this for each object that you estimated and measured. What do you notice?

Do you consistently *overestimate* or *underestimate*? Do you make better estimates on short things or on long ones?

Notice that if you estimated perfectly, you would place a dot on the diagonal line.

How many objects did you estimate very closely?

Write a paragraph in your journal explaining how you interpret your graph according to your estimation skills. Indicate how you might evaluate your estimation skills for longer lengths (e.g., the width of your classroom, the length of the chalkboard).

MEASURING AROUND THE HOME

A. Use your folding meterstick (or metric tape or ruler) to measure these objects in your home after you first have estimated their lengths.

	Estimate	Measurement
1. Height of a room from floor to ceiling	_____	_____
2. Width of a doorway	_____	_____
3. Diagonal length of a TV screen	_____	_____
4. Diagonal of a dinner plate	_____	_____
5. Length of a fork	_____	_____
6. Thickness of a telephone directory	_____	_____
7. Circumference of a small pot	_____	_____

B. Find something in your home that measures

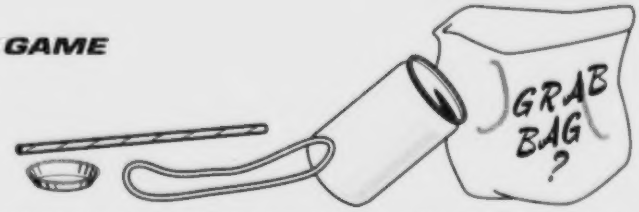
1. 1 cm ______________________________
2. 5 cm ______________________________
3. 10 cm ______________________________
4. 50 cm ______________________________
5. 100 cm ______________________________

Which was harder, A or B? _____ Which was more fun? _____ Which is more interesting, measuring pictures of objects or measuring real objects? ______________________________

C. Look for things in your home or in magazines or newspapers that are marked with metric measurements. List as many as you can.

ACTIVITY 2
ESTIMATE AND MEASURE GAME

Let's play a game.



Directions: On each round a player (selector) picks an object from the Grab Bag and specifies what is to be measured and in what unit (decimeter, centimeter, millimeter)—for example, the width of a paper clip in millimeters (mm), around a soda can in centimeters (cm), the length of a piece of string in decimeters (dm). On the score sheet (below) each player records the object, the unit, what is measured, and his or her estimate. Then the selector measures the object and reports the result. Players record this along with the difference between their estimate and the measurement. Another player then beomes the selector and specifies what is to be measured and in what unit. The game continues until all the objects in the Grab Bag are used, or for ten rounds. The player with the smallest total difference is the winner.

Finding the total difference will involve some conversion between units—for example, expressing all differences in millimeters. Determining the winner may also involve conversions—for example, who wins if Erich has 1248 mm, Jaime has 12 cm, and Kendra has 1 dm and 3 cm?

Score Sheet:

Object	Unit	What to measure	Estimate	Measurement	Difference

CHALLENGE PROBLEMS:

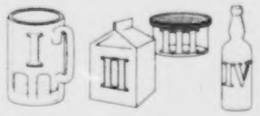
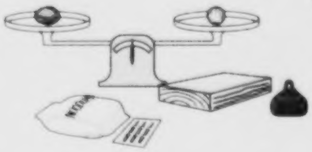
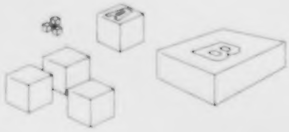
1. To challenge another group in your class, make up a set of cards that specify what is to be measured and in what unit for each item in a Grab Bag.
   - Make one set of cards for a Grab Bag containing items for measuring length.
   - Make one set of cards for a Grab Bag containing items for measuring capacity.
   - Make one set of cards for a Grab Bag containing items for measuring weight.

2. Which would you rather have, your weight in nickels or a crisp new $50 bill? ____________________

   Explain. ____

ACTIVITY 3
ESTIMATION FAIR

You will need a notebook (or something to write on) and a pencil. For each activity, estimate before you measure. Activities 1–8 can be done in any order—to save time, go to whichever activity has the fewest people waiting. When you have finished the eight activities, put your entries on the two graphs of item 9.

		Estimate	Actual	Difference
	1. Liquid Measurement A. Which holds the most, I, II, III, or IV?		X	X
	B. How many **liters** does it hold?			
	C. How many **milliliters** does it hold?			
	2. Weight—Which is heavier? A. Walnut or marble?		X	X
	B. Sinker or block?		X	X
	C. Needles or Noodles?		X	X
	D. Estimate the weight in grams of one item and weigh it on the scale.			
	3. Area—What is the area… A. …of the coaster in **cm²?**			
	B. …of the polygon in **dm²?**			
	4. Body Measurements A. How many **centimeters** *tall* are you?			
	B. How many **centimeters** long is your waist *or* your foot?			
	5. Sponge Throw How many **meters** can you throw a wet sponge?			
	6. From Here to There How many **meters** is it from A to B?			
	7. In a Minute—How many times… A. …can you jump rope?			
	B. …can you bounce a ball?			
	C. …can you touch your toes?			
	8. Volume A. How many **cm³** fill Box A?			
	B. How many **dm³** fill Box B?			
	C. Estimate the volume of Box B in **cm³**.			

9. Graphs
   - A. Record your favorite estimation activity on graph A.
   - B. Record the activity in which your estimate was closest to the actual measurements on graph B.

ACTIVITY 4
MEASURING ME

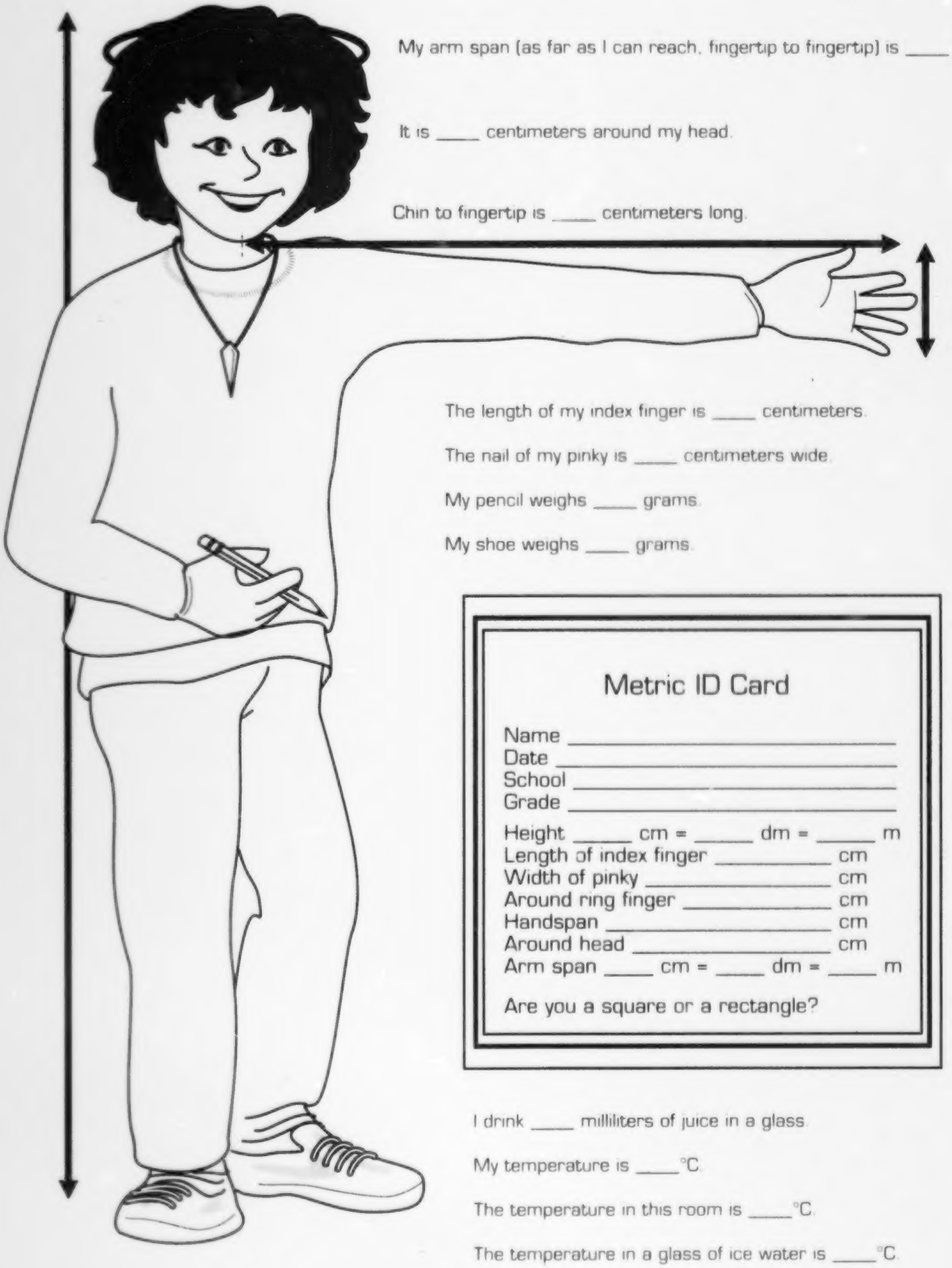
My height is ____ centimeters.

My schoolbag weighs ____ grams.

My arm span (as far as I can reach, fingertip to fingertip) is ____ centimeters.

It is ____ centimeters around my head.

Chin to fingertip is ____ centimeters long.



My handspan is ____ centimeters.

The length of my index finger is ____ centimeters.

The nail of my pinky is ____ centimeters wide.

My pencil weighs ____ grams.

My shoe weighs ____ grams.

Metric ID Card

Name ______________________________
Date ______________________________
School ______________________________
Grade ______________________________

Height _____ cm = _____ dm = _____ m
Length of index finger __________ cm
Width of pinky ________________ cm
Around ring finger _____________ cm
Handspan ____________________ cm
Around head _________________ cm
Arm span ____ cm = ____ dm = ____ m

Are you a square or a rectangle?

I drink ____ milliliters of juice in a glass.

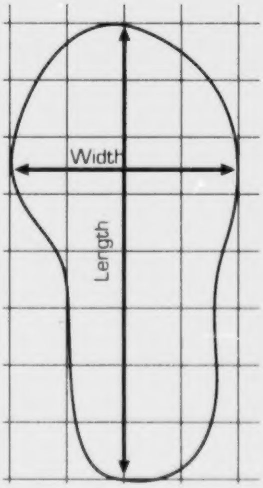
My temperature is ____°C.

The temperature in this room is ____°C.

The temperature in a glass of ice water is ____°C.

ACTIVITY 5
HOW BIG IS YOUR FOOT?

Problem: How can we measure the area of an irregular region?



1. At the right is a tracing of the foot of a one-month-old baby on centimeter grid paper.

   How long is the foot (toe to heel)? ______ cm

   How wide is the foot? ______ cm

   How many little squares does the foot cover? ______

   Count them, estimating the total of all the fragments of squares. About _____ squares

2. *a.* Trace your own foot on centimeter grid paper and find the same measurements.

   Length ______ cm

   Width ______ cm

   Number of squares covered ______

   b. How many times longer is your foot than the baby's foot? _____

   How many times wider is your foot than the baby's foot? _____

   How many times more squares does your foot cover than the baby's foot? _____

   c. How does your foot compare to the baby's foot? Does the number of squares covered increase by the same factor as the length or the width? Explain.

3. Trace an adult's (perhaps a parent's) foot on centimeter grid paper and make the same measurements. How does the size of the adult's foot compare with the baby's foot? With your foot?

CHALLENGE PROBLEM:

Draw a circle with a radius of 7 cm on centimeter grid paper. Draw a square around the circle so the four sides of the square just touch the circle.

a. What is the area of the square? _____ square centimeters (cm^2)

b. What is the area of the circle? (Count the total of all the fragments of squares covered by the circle.) _____ square centimeters (cm^2)

c. Approximately what fractional part of the square does the circle cover? _____

Repeat the same procedure and find the results for a circle with a radius of 5 cm; 6 cm.

On the basis of your data, what conjecture might you make comparing the area of a circle with the area of a square surrounding it?

ACTIVITY 6
AVERAGE NTH GRADER (N = 5, 6, 7, 8)

What does the average *N*th grader look like?
What are his or her favorite sports? Music? Books? Foods?
How does the average *N*th grader spend her or his time?
What does "average" mean?
What data does your group need to gather in order to answer these questions?

Perhaps you need to take a survey. Before beginning, certain decisions have to be made by your group, such as the following:

- Define what is being investigated.
- What data need to be collected?
- How will the data be collected and from whom?
- How will the data be organized and analyzed? What comparisons will be made?

After you have examined the data, identify patterns. Use the evidence to formulate conjectures and theories. Also view the data from different perspectives in order to address other questions. When you have finished, prepare a detailed report of your project's investigation including graphs and charts, as well as conjectures and ideas on how the investigation might be extended.

Below is a sample survey form to give you some ideas.

Survey Form

Name (print) ______________________ Class __________ Date __________

My Measurements:

Date of Birth: Month ______ Day ______ Year ______ Age: ______ years and ______ months
Height: ______ cm
Cubit: (length from elbow to tip of longest finger) ______ cm
Handspan: Right hand ______ cm Left hand ______ cm
Stretched-out arm span: ______ cm
Length of pace: ______ cm Shoe/sneaker size (from heel to toe): ______ cm
Around head: ______ cm Width of smile: ______ cm
Waist size: ______ cm Length from waist to floor: ______ cm
Weight: ______ kg

My Favorites:

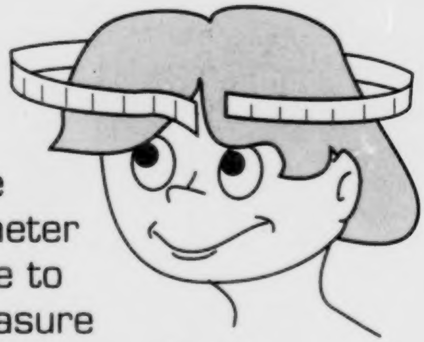
Favorite school subject: ______________________
Favorite food: ______________________
Favorite color: ______________________
Favorite sport: ______________________
Favorite type of music: ______________________
Favorite magazine or book: ______________________
Favorite TV show: ______________________
Favorite movie star: ______________________

How I Use My Time:

Amount of time I usually spend on homework each day: ______________________
Amount of time I usually spend watching television each day: ______________________
Amount of time I usually sleep each day: ______________________
What I most like to do in my spare time: ______________________

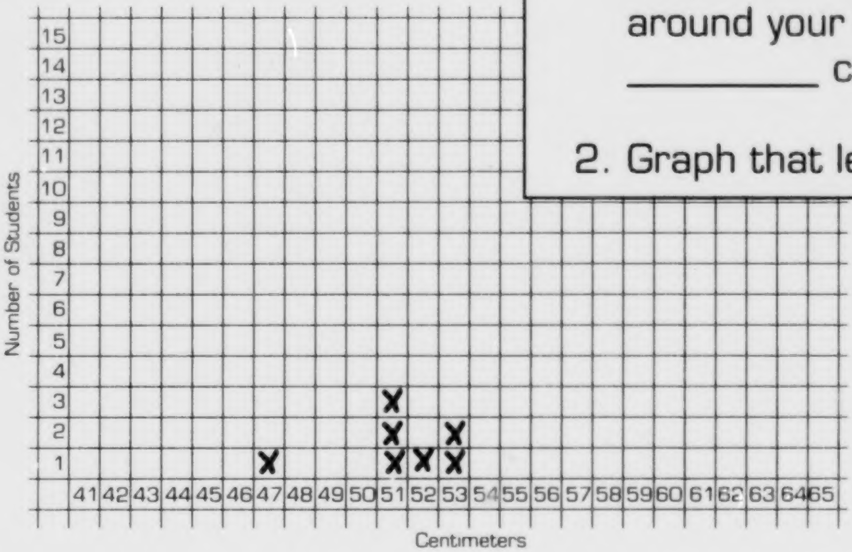
ACTIVITY 7A
LENGTH LEARNING CENTERS

Around Your Head



1. Use a meter tape to measure around your head. ________ cm

2. Graph that length.



How many centimeters around your head?

How do you think your class graph of head sizes might compare to a graph made of your class's parents' head sizes? What might be different? What might be the same?

Do older people have larger head sizes?

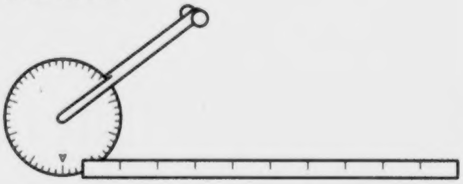
Do taller people have larger head sizes?

Investigate these questions by taking a survey and report the results.

To what group of people would knowing the span of head sizes of people be important? Explain.

ACTIVITY 7A (CONTINUED)
LENGTH LEARNING CENTERS

Metersticks and Trundle Wheels



1. First estimate. How many meters are from the door of your classroom to the door of the classroom next door? _____m

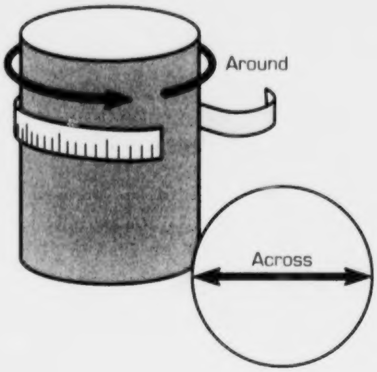
   Now use a meterstick: _____ m

2. Use a trundle wheel. How many meters from your classroom to ________? ______m

3. Which would you use to measure the length of the hall, a meterstick or a trundle wheel?

   _________________________ Why? ______________Do it: _______m

Around and Across



A. Estimate the length around the object in millimeters: ______mm

   Now measure around the object in millimeters.
   Around: ______mm

B. Measure across the object.
   Across: ______mm

C. Now divide: around ÷ across.
   What is the result? _______ (Quotient)

D. Record your results in the table for this activity.

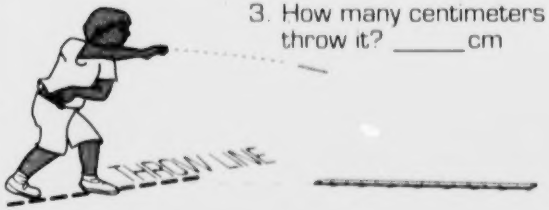
Object	Around	Across	Quotient

E. Try this again with another round object. Record the results above. How do your results compare with those of a classmate?

Metric Olympics

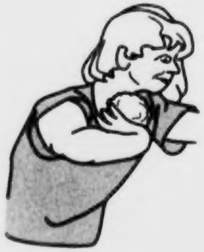
Event: Javelin Throw

1. Stand at the throw line.
2. Throw the straw as far as you can.
3. How many centimeters did you throw it? _____cm



Metric Olympics

Event: Cotton Ball Shotput



1. Place a cotton ball in your hand and raise it to your shoulder.
2. Thrust hand forward and release the cotton ball.
3. How many centimeters did you toss the cotton ball?

_____cm

Metric Olympics

Event: Standing Long Jump

1. Stand on the line. (Toes must be behind the line.)
2. From a standing position, jump forward.
3. How far did you jump?

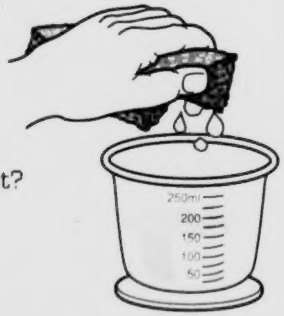
_____cm

Metric Olympics

Event: Sponge Squeeze

1. Dip a sponge in water.
2. Squeeze the water into the container.
3. How many milliliters of water can you squeeze out?

_______ mL



ACTIVITY 7C
CAPACITY LEARNING CENTERS

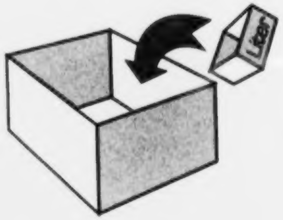
Liters

1. Estimate how many liters fill up the large box.

   Guess: ______ L

2. Use the liter box to check your guess.

   Measure: ______ L



Guess and Measure

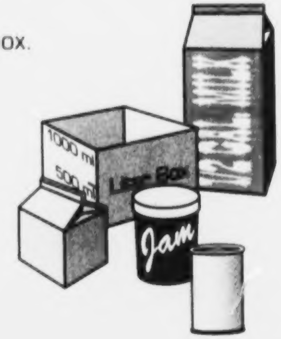
1. Pick a container. Compare it to the liter box.

   Guess how many milliliters it holds.

   ______ mL

2. Now measure how much it holds.

   ______ mL



Fix That Leak

1. How many drops make 1 milliliter? Use an eyedropper to check.

2. A faucet drips 1 drop every second. How many milliliters of water are wasted in a day?

   ____ mL, or about ____ L

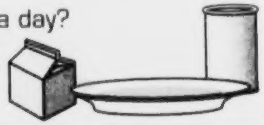
How Much Do You Drink in a Day?

1. Measure how many milliliters the paper cup holds. ______ mL

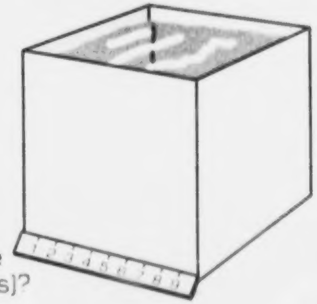
2. About how many of these cups of liquids do you drink in a day? (For example: milk—3 cups; water—2 cups;...)

3. How many mL do you drink in a day?

   ______ mL, or about ______ L

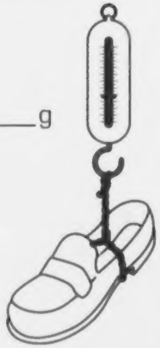


Weighing Water with a Ruler



1. What is the volume of the box?
   _____ cm^3
2. How many mL of water does the box hold?
   _____ mL of water
3. What is the weight of the water in the box (in grams)?
   _____ g

How Much Does Your Shoe Weigh?



1. My shoe weighs _____ g.
2. Use the results of part 1 to find the total weight of your pair of shoes. _____ g
3. Does your pair of shoes weigh more or less than 1/2 a kilogram? _____

Guess the Weight

1. Pick a container and compare its weight with the 1-gram (small cubeogram) or 50-gram weights.

Cereal
Candy

Guess its weight. ____ g

2. Now check the bottom of the container for the actual weight.
   _____ g
   How far off was your guess?
   _____ g
3. Try with another container.
   Guess: _______ g
   Weight: ______ g
   Off by: _______ g

Recycle Aluminum

1. What is the weight of 1 aluminum can?
   About _____ g
2. How many cans make a kilogram?
   About _____ cans
3. Suppose that each student in your class collected 10 cans each week for the whole school year (35 weeks). How much money would the class earn in all if you got 50¢ per kilogram of cans? $ ______

ACTIVITY 8
METRIC MODELS AND PROJECTS

With each metric measurement given in column A, match the best physical model of that unit as described in column B.

Column A

mm ________
cm ________
dm ________
m ________
km ________

dm^2 ________
m^2 ________

cm^3 or cc ____
15 mL ______
L ________

mg ________
g ________
kg ________

0°C ________
20°C ______
40°C ______

Column B

a. weight of a nickel
b. thickness of a dime
c. a little more than a quart
d. a little more than 2 pounds
e. height of a doorknob from the floor
f. size of a piece of toast
g. width of a small fingernail
h. weight of a paper clip
i. width of a clenched fist
j. a little longer than 1/2 mile
k. a little larger than the top of a card table
l. capacity of a tablespoon
m. weight of a little more than a quart of water
n. capacity of a measuring cup
o. approximately 20 short city blocks
p. weight of a few grains of sand
q. normal room temperature
r. normal body temperature
s. width of an adult's handspan
t. weight of a newborn baby
u. a record-breaking heat wave
v. size of a man's handkerchief
w. freezing point of water
x. capacity of an eyedropper

METRIC QUIZ

Consider each of the following sentences and indicate by a check (✓) which are *likely* to be true and which are *unlikely* to be true.

Likely Unlikely

1. A basketball player is 3 meters tall.
2. She ran 1 kilometer in a minute.
3. A bicycle was traveling at 12 kilometers an hour.
4. He planted a tulip bulb in 1 cubic centimeter of dirt.
5. He bought a meter of milk at the grocery store.
6. They made ice cream in a freezer set at 10° Celsius.
7. A professional football player weighs 120 kilograms.
8. A top of a card table was covered with a square meter of cloth.
9. The temperature dropped to 25° Celsius and it began to snow.
10. She found a pencil 1 milligram long.
11. His handspan measures 10 decimeters.
12. The area of the usual postage stamp is 20 square centimeters.
13. A gram is the same as a milliliter.
14. A milliliter is the same as a cubic centimeter.

For each of the statements above you have checked (✓) *unlikely*, write one or two sentences in your journal to explain (justify) your conclusion.

ACTIVITY 8 (CONTINUED)
METRIC MODELS AND PROJECTS

METRIC GROUP PROJECTS

1. Design a rabbit hutch or a doghouse that can be made from a sheet of plywood that measures 1 meter by 2 meters. Make the hutch or doghouse as large as possible and show how the pieces can be laid out on the plywood before cutting.

2. Design and implement an investigation to answer these questions: "How much water do you use each day?" and "How much water does your class use in a typical day?" (Do other classes in your school agree with the results in your class?) (See Mathematics Resource Project [1978], pp. 223–24.)

3. Design an experiment to determine how much water is wasted by a leaky faucet in an hour, a day, a week, a month, a year. (See Mathematics Resource Project [1978], p. 225.)

4. Your class has decided on a paper-recycling project for your school. Some students have found that recyling companies pay according to the type of paper (e.g., newspaper, bond paper, computer paper). In order to plan for collecting, packing, and shipping the paper, you need to gather more data. How should you begin? Describe what information and data you need and how you will organize this paper-recycling project.

CHALLENGE PROBLEMS:

1. A company delivers 100 cartons of paper to your school. Each carton measures 40 cm by 30 cm by 25 cm. The paper is to be stored in a space 2 m by 1.5 m by 2.5 m. Is it possible to store all the cartons of paper in this space? Explain.

2. How much does a weekday newspaper weigh? ____ g. Suppose each student in your class collected 5 daily newspapers, Monday through Friday, each week for a whole year. You get 4 cents per kilogram to recycle the paper. How much would your class of 25 students earn in one year? Explain.

3. *a.* The gas tank in your family car holds 60 liters. If your car averages 12 kilometers per liter in non-city driving, approximately how far could your family drive in the country before needing to refill the gas tank? Explain.

   b. Suppose you want to travel 500 kilometers and you get 12 kilometers to the liter, but you are low on money and want to buy just enough whole liters to make the trip. How many liters should you buy? Explain.

CLUSTER B
DISCOVERING PATTERNS AND LOGICAL CONNECTIONS TO CREATE MEASUREMENT FORMULAS

Activity 9: PeRIMeters and Areas of Figures
  a. Rectangles and Squares
  b. Parallelograms and Triangles
  c. Trapezoids
  d. Family Trees

Activity 10: Geoboard Activity—PeRIMeter, Area, and Pick's Theorem

Activity 11: Squares and the Pythagorean Theorem

Activity 12: Measuring a Circle

Activity 13: Computer Investigation: Measurements of Similar Figures

Activity 14: Surface Area of Polyhedra and Spheres

Activity 15: Volume of Polyhedra and Spheres

OBJECTIVES OF CLUSTER B:

1. To explore and develop conjectures (generalizations or formulas) for finding the peRIMeter, area, and volume of geometric figures through hands-on activities
2. To apply reasoning skills—inductive, deductive, proportional, and visual reasoning (as appropriate)—in problem explorations involving measurement
3. To explore concepts of length and area using a geoboard; to investigate, find patterns, and make discoveries and conjectures about the interrelationship of the number of boundary points and interior points of a geoboard figure to its area (i.e., Pick's theorem)
4. To discover on a geoboard the lengths of line segments (e.g., $\sqrt{2}$, $\sqrt{5}$) that cannot be represented by rational numbers; to discover the Pythagorean theorem
5. To investigate, using computer software, the relationships of lengths, perimeters, and areas of similar figures
6. To make conjectures and give informal arguments to justify conjectures (measurement formulas) and to explain interrelationships among the formulas (family trees)
7. To understand when and how the conjectures and generalizations developed are used in solving real-world measurement problems

TEACHING NOTES FOR CLUSTER B

Reasoning is essential to knowing and doing mathematics. Reasoning is the major focus of the activities in this cluster. As students formulate conjectures and demonstrate the logical validity of their conjectures, they are involved in creative acts of doing mathematics.

> ***"A classroom that values reasoning also values communicating and problem solving. . . . A climate should be established in the classroom that places critical thinking at the heart of instruction."* (Curriculum and Evaluation Standards [NCTM 1989, p.29])**

Activity 9a–d. PeRIMeters and Areas of Figures

Materials: Copies of Activity Sheets 9*a–d* for each student, centimeter rulers, packet of MAP shapes (see Appendix), L-square, transparent centimeter grid, scissors, tape

◆ ◆ ◆ ◆ ◆ ◆ ◆ ◆

The completion of this group of activities might extend over several days. The focus is on intuitive (hands-on, visual, inductive) and logical (informal deductive) arguments to justify the generalizations (formulas) for finding areas of shapes. Since the activities are developed within the context of a City Map, a concrete set of shapes, called MAP (map area parts), is provided so that students can explore various ways of finding the areas of different-shaped plots of land. It is important that the concept of the height of a figure be developed through the use of an L-square. An effort should be made to have the students think of the area of a rectangle as base times *height* rather than length times *width*. (This will deal with the problem of students later computing the area of a nonrectangular parallelogram by finding the product of two adjacent sides.) Initially students should be encouraged to find the area of a rectangle by counting the number of square units and be able to explain and justify the generalization of finding the area by computing the product of base and height. The students should develop their conjectures for finding the areas of figures by using the concrete set of MAP shapes (see Appendix) and be able to give informal logical arguments to justify their conjectures.

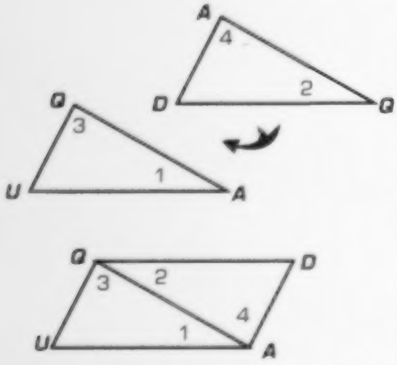
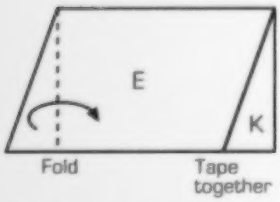


Fig. 12

Example: It is not enough to say the area of a triangle is 1/2 the area of a parallelogram because it looks like it. Rather, the student might give an informal argument, pointing out that the two triangles in figure 12 are congruent (because they fit together exactly) and form a parallelogram. $\overline{QD}$ is parallel to $\overline{UA}$ because angle 1 ≅ angle 2, and $\overline{QU}$ is parallel to $\overline{DA}$ because angle 3 ≅ angle 4; so the sides of the figure are parallel and therefore the figure is a parallelogram; so the area of one triangle is 1/2 the area of the parallelogram, that is, the area of a triangle = $(1/2)b \times h$.

Students should be encouraged to find different ways of finding areas of figures using their models and then give informal arguments to justify their conclusions. For other approaches to finding the areas of these shapes, see Fuys, Geddes, and Tischler (1988). Visual arrangements give insight into a general argument and go beyond the particular numbers chosen. In many areas of mathematics we can look for such insightful visual reasoning. See "Proof without Words" in Maletsky (1987). Activity Sheet 9*d*, Family Trees, encourages students to reflect on relationships among the various area conjectures (formulas). For additional material on "family trees" and properties of shapes of figures, see Geddes (1992).

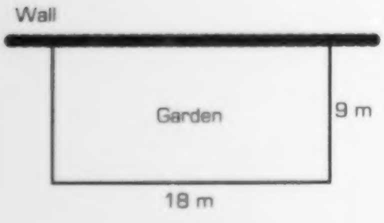


MAP shape K in the Appendix can be used to convert a nonrectangular parallelogram to a rectangle by attaching K to E with tape and then folding E on the dotted line.

Assessment—journal entry: Write a letter to a friend describing (*a*) how you can use your MAP models to discover the areas of different shapes, or (*b*) how you can explain and justify your conjecture for finding the area of one of the shapes, or (*c*) how you can create a family tree showing relationships among area conjectures.

Answers to Challenge Problems:

9*a*. Have students try different measures to find the largest area. Largest area = $18 \times 9 = 162$ m^2.



(Problem 9a)

9*b*. Area of each end = 7cm^2; area of each rectangle = 64 cm^2; total surface area of wedge = 206 cm^2. Amount of cardboard needed for 100 wedge-shaped boxes = 2.06 m^2.

9*c*. (1) Area of *ABFE* = 200 m^2; area of *DCKL* = 300 m^2; area of each trapezoid = 250 m^2; total area = 1000 m^2.

(2) The student is correct. Area of triangle *QUR* = $(1/2)h \times (UR)$, but $UR = (UA + AR)$ or $(UA + QD)$; so area of triangle *QUR* = $(1/2)h \times (UA + QD)$, which is the same as the area of trapezoid *QUAD*.

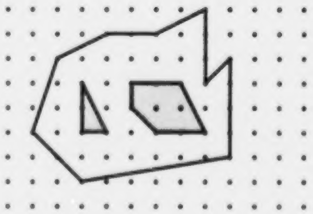
(3) The length of the midline is the average of the lengths of the two bases, or (1/2)(*UA* + *QD*). Thus, the area of *QUAD* = (1/2) · (*UA* + *QD*) × *h*, or midline × height.

Activity 10. Geoboard Activity—Perimeter, Area, and Pick's Theorem

Materials: Copy of Activity Sheet 10 for each student, geoboards with rubber bands, dot paper (for recording initial results)

Before beginning Activity 10, students should do some exploratory work on creating shapes on their geoboards. In the initial activities students find areas of shapes by subdividing the shape into square units or by building rectangles around the shapes (with rubber bands) and subtracting the area outside the shape from the area of the rectangle. As students explore the concept of "shearing," they discover that figures formed by shearing have the same base and equal heights and thus have equal areas.

Students investigate the area of shapes by considering the number of boundary and interior points. By starting the investigation with figures that have one interior point, students quickly discover the relationship between area and the number of boundary points. They recognize patterns in figures with more interior points that lead to the conjecture that Area = $(1/2)B + (I - 1)$, where B = the number of boundary points and I = the number of interior points (i.e., Pick's theorem for geoboard figures). Students are challenged to extend their investigation to figures that have holes and find that Area = $(1/2)B + (I - 1) + H$, where H = the number of holes, where the holes are distinct, not sharing any boundary points with each other or with the outer shape. *Note:* There needs to be some discussion of the meaning of boundary points and interior points for figures with holes (see fig. 13). For software illustrating an electronic geoboard, see Elastic Lines in the Bibliography.



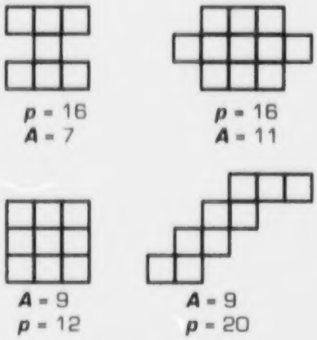
$B = 16 + 4 + 7 = 27$
$I = 19$
$A = 33\ 1/2$
$H = 2$

Fig. 13

Assessment: Create several geoboard figures and challenge your peers to find the perimeters and areas of these figures without using Pick's theorem. Check their area results by using Pick's theorem.

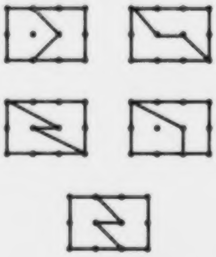
Answers to Challenge Problems:

1. Perimeter (p) of C = 16; Area (A) of C = 9. See other combinations of area and perimeter at the right.
2. The number of boundary points of this entire figure consists of the number of exterior points (16) of the figure plus the number of boundary points (4 + 7) of the holes. Thus, there are 27 boundary points and 19 interior points. The interior points of the holes are not counted, since they are not interior points of the given figure. The area of the given figure equals 33 1/2 square units. This is consistent with the extended formula described above where $H = 2$ in this case.
3. Other ways of dividing this rectangle are shown at the right.



$p = 16$, $A = 7$
$p = 16$, $A = 11$
$A = 9$, $p = 12$
$A = 9$, $p = 20$

(Problem 1)

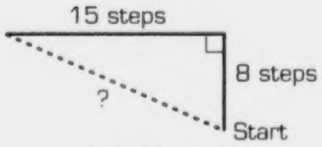


(Problem 3)

Activity 11. Squares and the Pythagorean Theorem

Materials: Copy of Activity Sheet 11 for each student, geoboards with rubber bands, dot paper (for recording initial results), L-square (to check for right angles), calculators with square root key

In the initial investigation of squares, students find (by subdividing the figure into square units) that the area of a square is equal to the square of the side; by reversing the process, they should discover by "unsquaring" (taking the square root of the area) that they can find the length of the side. For example, if the area of the square is 2, the side is $\sqrt{2}$. Have students build squares with sides b and c on their geoboards and find their areas; they soon discover that $b = \sqrt{2}$ and $c = \sqrt{5}$. The use of the square root key on the calculator should be discussed along with the concept of irrational number, noting that geoboard lengths of oblique line segments on the geoboard can represent irrational numbers (e.g., $\sqrt{2}$, $\sqrt{5}$, $\sqrt{8}$). In the second investigation, students discover the Pythagorean relationship in terms of building squares on the sides of right triangles. For an excellent videotape on the Pythagorean theorem, see Apostol (1988).



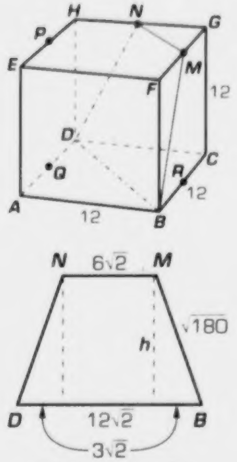
(Problem 1)

Assessment—journal entry: Write several paragraphs describing the Pythagorean theorem, when and how it can be used, and explaining why you think it is so important and famous.

Answers to Challenge Problems:

1. The distance from the starting point is 17 steps by using the Pythagorean theorem.

3. *a.* Area of square $MPQR = 12 \times 12 = 144$ cm^2.
   b. Area of rectangle $EACG = 12 \times 12\sqrt{2} = 144\sqrt{2}$ cm^2.
   c. Area of equilateral triangle $BEG = 288\sqrt{3}$ cm^2.
   d. Area of isosceles trapezoid $MNDB = 162$ cm^2 when $DB = 12\sqrt{2}$, $NM = 6\sqrt{2}$, $MB = \sqrt{180}$, height $(h) = 9\sqrt{2}$.



(Problem 3)

Activity 12. Measuring a Circle

Materials: Copy of Activity Sheet 12 for each student, collection of round objects (e.g., different sizes of cans, spools of thread, wheels), centimeter rulers, metersticks, centimeter grid paper, calculators

Groups of students should measure a diameter (a line segment that passes through the center of the circle and whose endpoints lie on the circle) and the circumference of five or six objects and record these data and the ratio *Circumference:diameter* in the chart. Data from the class should be assembled and patterns discussed, noting that the ratio $C:d$ is a little more than 3 (i.e., 3+). If some entries are less than 3, possible reasons might be discussed. There is no need to introduce the symbol π (pi) immediately. Try to build insight and intuition by having students physically demonstrate with string that the length of the diameter of an object fits around the circumference of the object a little more than three times, that is, $C = (3+)(d)$; students should begin to appreciate this remarkable phenomenon—that for any circle in the world, $C:d = 3+$. (Some early biblical references to this relationship in the building of Solomon's temple might be cited: 1 Kings 7:23 and 2 Chronicles 4:2.)

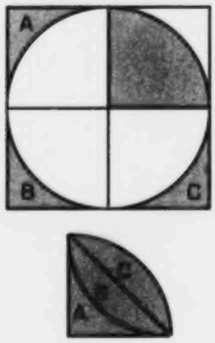


Fig. 14

The experiment to find the area of a circle by counting square units (see the Challenge Problem of Activity 5) or by circumscribing a square around the circle leads to the conjecture that the area of a circle is a little more than the areas of three equal squares whose side equals the radius of the circle; thus Area of a circle = $(3+)(r^2)$. One can demonstrate this physically on the overhead projector by cutting the shaded portion of the circle in figure 14 into three pieces and then fitting the three pieces into the three corners. *A* fits exactly; *B* and *C* are a bit too large. Thus Area of a circle = $(3+)(r^2)$.

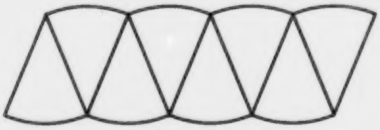


Fig. 15

One Challenge Problem is to find the area of a circle another way by cutting a circle into congruent sectors and rearranging the pieces to form a shape resembling a slightly scalloped parallelogram (see fig. 15). The area of a parallelogram is base times height; here the base approaches one-half the circumference of the circle, and the height approaches the radius of the circle; thus Area = $(1/2)C \times r$, yielding $(3+)(r^2)$.

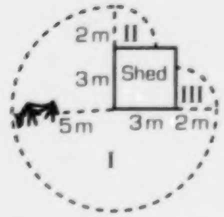
◆ ◆ ◆ ◆ ◆ ◆ ◆ ◆

Assessment—journal entry: Write a short essay or poem on "Circles and Civilization" or on "All I Know about a Circle."

Answers to Challenge Problems:

1. Area of "scalloped" parallelogram (see fig. 15) = $(1/2)C \times r = 1/2 \cdot 2\pi r \cdot r = \pi r^2$.
2. Area of circle surrounding station WSEE with radius of 50 km is 2500π km^2.
3. Mooey has a grazing area of 25π m^2 when tied to the post. When tied to the corner of the shed, Mooey's grazing area is $(83/4)\pi$ m^2, or approximately 65 m^2.

The common error made by students in solving this problem is to find the area of a circle with radius 5 m and then subtract the area of the shed, thus not recognizing the constraints on Mooey's rope as it goes around the shed.



Area I $= (3/4)\pi \cdot 5^2 = (75/4)\pi$ m^2
Area II $= (1/4)\pi \cdot 2^2 = 1\pi$ m^2
Area III $= (1/4)\pi \cdot 2^2 = 1\pi$ m^2

(Problem 3)

Activity 13. Investigation: Perimeters and Areas of Similar Triangles

Materials: Copy of Activity Sheet 13 for each student, Macintosh- or IBM-compatible computers, software such as the Geometric Supposer or Euclid's Toolbox (see Bibliography).

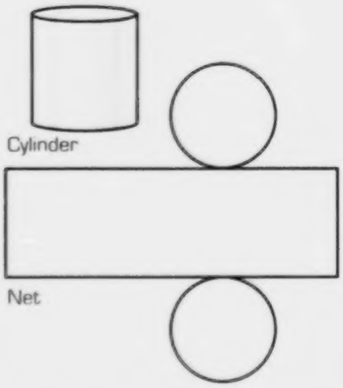
The use of geometry exploration software (e.g., the Geometric Supposer) allows students to make and test conjectures about geometric figures. Euclid's Toolbox software is user-friendly and allows students to explore geometry, collect data, make conjectures, and test hypotheses. Students can construct a shape and measure its attributes; then with one keystroke, they can repeat the same measurements with a shape of a different size. Similar types of activities can also be done using the Geometer's Sketchpad (see Bibliography) on a Macintosh computer using dilations to create similar triangles. In this investigation students will discover that the ratio of the areas of similar triangles is the same as the ratio of the squares of their corresponding sides. They should be encouraged to extend their investigation to include other similar shapes; see Activity 20*b*, Grid Enlargements.

Assessment: Prepare a report on your computer investigation; describe your data collection, analysis, conjectures, and justification of conjectures concerning the perimeters and areas of similar figures. Indicate how you might extend your investigation.

Activity 14. Surface Area of Polyhedra and Spheres

Materials: Copy of Activity Sheet 14 for each student, several boxes in the form of rectangular prisms, several cans (right circular cylinders) with labels, globe (ball), oranges, waxed paper, knife; models of prisms, pyramids, cylinders, cones, spheres

Students engage in hands-on approaches to find the measures of the surface area of a rectangular prism (Area = $2lw + 2lh + 2wh$), a right circular cylinder (Area = $2\pi rh + 2\pi r^2$), and a sphere (Area = $4\pi r^2$) and then develop and justify their conjectures. Students are frequently surprised to see the net of a right circular cylinder and to discover that the lateral area of the cylinder is equal to the area of a rectangle (the label), where the length of the rectangle is equal to the circumference of the base of the cylinder. To find the surface area of a sphere, the development is done empirically, since a mathematical proof requires more background than students have at this level. The "orange" experiment to find the surface area of a sphere, although a bit messy, develops considerable student insight. An alternative approach might be to make four



paper great circles of a sphere (ball) and have students cut up the paper circles to "wallpaper" the sphere in order to answer the question, How many paper great circles of a sphere are needed to cover the surface of the sphere?

Answers to Challenge Problems:

2. There is not enough material, without patching, to cover the two lampshades, since the circumference of the base of each shade is 40π cm, or approximately 156 cm, or 1.56 m.

3. Find a length of string that fits around your globe at the equator; call this length *e*. Now place this string around a great circle going through Hong Kong and New York City and measure on the string the distance in centimeters between the two cities; call this distance *d*. Then set up a proportion to find the actual distance in kilometers between the two cities, that is,

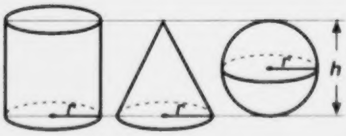
$$\frac{d\text{ (in cm)}}{e\text{ (in cm)}} = \frac{?\text{ (km)}}{40\,000\text{ km}}.$$

Substitute your values for *d* and *e* and solve the proportion.

Activity 15. Volume of Polyhedra and Spheres

Materials: Copy of Activity Sheet 15 for each student; collection of centicubes, Sage Kit (or comparable models of a prism, pyramid, cylinder, cone, and hemisphere) for each group; set of a cylinder, a cone, and a sphere that all have the same diameter and the same height; stacks of index cards, stacks of crackers (each stack having a different shape of cracker); colored water (or sand or rice) for pouring

Students should have some measurement experiences comparable to those in Activity 14 before engaging in this activity. The volume formulas discussed in this activity are developed empirically, since mathematical proofs of these formulas require more backgroud in mathematics than students have at this level. However, the empirical approach gives students some insight into the reasonableness of the generalizations. This activity requires hands-on work pouring water using the models of prisms, pyramids, cylinders, cones, and hemispheres.



The activities should be done in groups of four to six, keeping careful records of results, or Activity Sheet 15 can be used as a script to use with the whole class where the teacher and student assistants explore and demonstrate relationships between and among the shapes. Students should be encouraged to make estimates before measuring. After the explorations and demonstrations, students should be able to explain their conjectures, namely that the volume of a prism (or cylinder) is the product of the area of its base (b) and its height (h), that is, $b \times h$, and that the volume of a pyramid (or cone) is one-third the product of the area of its base (b) and its height (h), that is, $1/3 \times b \times h$. In the experiment using models of a cylinder, a cone, and a sphere having the same radius (r) and the same height (h), where $h = 2r$, students should discover that the water displaced by the sphere fills the cone *twice*. This leads them to the discovery that since the volume of the cylinder is $\pi r^2 h$ and the volume of the cone is $(1/3)\ \pi^2 h$, the volume of the sphere is $(2/3)\ \pi r^2 h$; but since $h = 2r$, the volume of the sphere with radius r is $(4/3)\pi r^3$. Students should be able to justify the final conjecture of each experiment and to apply each conjecture to real-world problem situations.

◆ ◆ ◆ ◆ ◆ ◆ ◆ ◆

In their investigations students will discover that the ratio of the volumes of two similar shapes is the same as the ratio of the cubes of their corresponding parts; see Activity 20*b*, Building Staircases. A final challenge is to have students create a "family tree" to show the ancestors and relationships among the formulas for the volumes of prisms, cylinders, pyramids, cones, and spheres and discuss how this tree relates to the "family tree" created in Activity 9*d*.

Assessment: Select two or three of the Challenge Problems from Activities 9–15. For each problem, describe how you thought about it; your approaches and strategies; obstacles encountered; partial, tentative, or complete solution(s); and how you think the problem might be extended.

Answers to Challenge Problems:

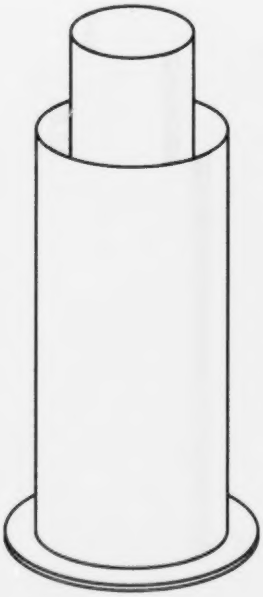
1. For 20 cents, you get approximately 180 cm^3 of orange, so for 1 cent, approximately 9 cm^3. For 30 cents, you get approximately 382 cm^3 of orange, so for 1 cent, approximately 12.7 cm^3. The 30-cent oranges are clearly the better buy.

2. The volume of the unfilled portion of the glass is approximately 80 cm^3. The volume of the ball is approximately 66 cm^3. The water will not overflow if the ball sinks.

3. The volume of one giant scoop is approximately 268 cm^3. The volume of each cylindrical container is approximately 8164 cm^3. So there are about 30 giant scoops of ice cream in each container.

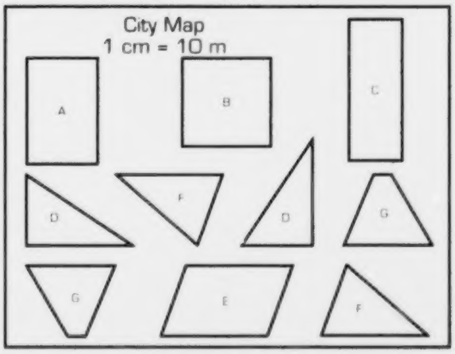
4. Since the mini and regular doughnuts are similar in shape, their volumes can be compared by using the principle that volumes of similar shapes are to each other as the cubes of their corresponding parts, that is, Volume of mini:volume of regular = (width of ring of mini)^3:(width of ring of regular)^3, where the ring width of the mini is 2.25 cm and ring width of the regular is 3 cm. When compared, the volume of the minis is less than one-half the volume of the regular doughnuts. So are the minidoughnuts really a bargain?

5. Find the volume of each right circular cylinder formed by rolling one sheet around the length of the paper and the other around the width by using the formula that the volume of a right circular cylinder = area of base × height, or $\pi r^2 \times h$. If 8 1/2 × 11 paper is used, the volume of the cylinder with height of 8 1/2 is approximately 82 cubic units and the volume of the cylinder with height of 11 is approximately 63 cubic units. Clearly, the two volumes are not equal, which surprises the students. One great way to demonstrate this inequality is to place the taller cylinder inside the shorter one, standing on a flat plate. Fill the tall one with popcorn, then pull out the tall cylinder slowly so its contents goes into the shorter cylinder. Is the shorter cylinder filled? No—much to the students' amazement. (*Note:* In this problem, the sheet of paper referred to is measured in inches.)



(Problem 5)

ACTIVITY 9A
PERIMETERS AND AREAS OF SQUARES AND RECTANGLES



Note: Scale of 1 cm:10 m refers to MAP scale models.

Your group (representing a company of landscape architects) has been hired by the city to landscape a downtown area containing different sizes of plots of land as shown in the City Map at the right. Before proceeding on the job, your group realizes that you need to know the area of each of these plots of land. How can you find this information from the MAP scale models (scale 1 cm:10 m)?

What is the most specific geometric name for each MAP shape?

A ______________ B ________________
C ______________ D ________________
E ______________ F ________________
G ______________

Use the set of MAP scale models, a centimeter ruler, and an L-square (or centimeter grid) to find the perimeter and area of MAP shapes A, B, and C. Which is largest, A or B or C, or do they have the same area? Explain.

Investigation: Complete the following:

MAP shape A (model): length (l) = ___; base (b) = _____; perimeter = _____; area = _____;
width (w) = ____ ; height (h) = ____
plot A: actual perimeter __________; actual area ____________

MAP shape B (model): length (l) = ___; base (b) = _____; perimeter = _____; area = _____;
width (w) = ____ ; height (h) = ____
plot B: actual perimeter __________; actual area ____________

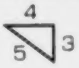
MAP shape C (model): length (l) = ___; base (b) = _____; perimeter = _____; area = _____;
width (w) = ____ ; height (h) = ____
plot C: actual perimeter __________; actual area ____________

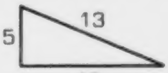
Conjecture: In terms of b and h, the perimeter of a rectangle is ______________
and the area of a rectangle is ___________________.

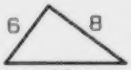
Conjecture: In terms of its side s the perimeter of a square is _______________
and the area of a square is ____________________.

If the perimeters of two shapes are equal, do they have equal areas? Explain and illustrate.
If the areas of two shapes are equal, do they have equal perimeters? Explain and illustrate.

Investigate with your MAP scale models how to find the area of MAP shape D. What is the length of its base (b)? ______ Its height (h)? _______ How is MAP shape D related to MAP shape A? Explain and justify your conclusion. What is the area of MAP shape D? ___________ Does your method for finding the area of MAP shape D work for other right triangles? Why or why not? Consider the right triangles below. Find the area of each and justify your conclusion.



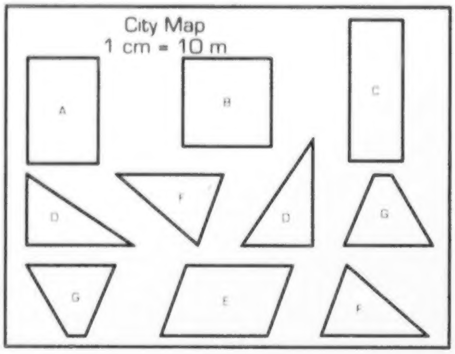




Conjecture: The area of a right triangle with base (b) and height (h) = ___________.

Challenge Problem: You have 36 meters of fencing and want to fence a garden in the shape of a rectangle against a wall. Draw a picture of the shape of the fenced garden with the largest area (give the dimensions).

ACTIVITY 9B
AREAS OF PARALLELOGRAMS AND TRIANGLES

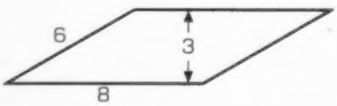


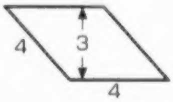
1. **Investigate** with your MAP scale models how to find the area of MAP shape E. Describe or illustrate several ways that your group explored for finding its area. Did anyone suggest the area was 102 cm² by multiplying 12 cm by 8.5 cm? Explain why this is not correct.

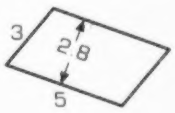
   What is the length of base (*b*) of shape E? __________
   What is the height (*h*) of shape E? __________
   How is shape E related to shape A? Justify your conclusion.

   What is the area of shape E? ___________ Explain why your method of finding the area of shape E is correct. Does your method for finding the area of shape E work for other parallelograms? Why or why not?

   Consider the parallelograms below. Find the area of each and justify your conclusion.







Conjecture: The area of a parallelogram with base (*b*) and height (*h*) = ____________.

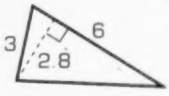
2. **Investigate** with your MAP scale models how to find the area of MAP shape F. Describe or illustrate several ways that your group explored for finding its area.

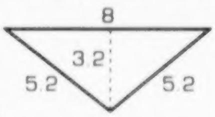
   What is the length of the base (*b*) of shape F? _________
   What is the height (*h*) of shape F? _________

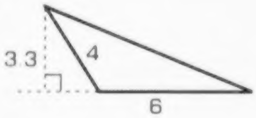
   How is shape F related to shape E? Justify your conclusion.

   What is the area of MAP shape F? ______________ Explain why your method of finding the area of shape F is correct. Does your method for finding the area of shape F work for other traingles? For example, obtuse triangles? Why or why not?

   Consider the triangles below. Find the area of each and justify your conclusion.



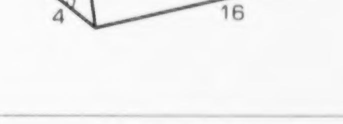




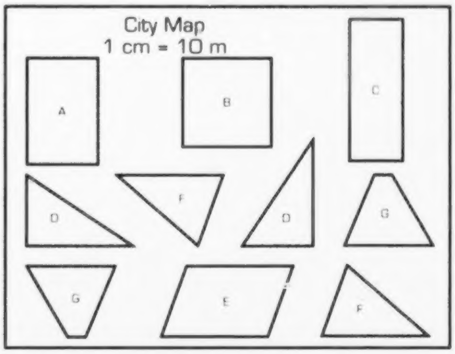
Conjecture: The area of a triangle with base (*b*) and height (*h*) = _____________.

CHALLENGE PROBLEM:

One manufacturer of chocolate bars packages her product in boxes that resemble triangular wedges (see diagram at the right). The two ends of the wedge are equilateral triangles with side = 4 cm and height = 3.5 cm. The other three sides of the wedge are rectangles 16 cm long. How many square meters of cardboard will be needed to produce 100 such wedge-shaped boxes?



ACTIVITY 9C
AREA OF A TRAPEZOID



Investigate with your MAP scale models how to find the area of MAP shape G. Notice that shape G has two parallel bases—a lower base (*b*) and an upper base (*b′*).

Describe or illustrate several ways your group can explore for finding the area of shape G, which is called a "trapezoid."

For example: Consider using your two congruent models of figure G and experiment putting them together in different ways so that two sides fit together exactly. What different shapes are formed?

Does one shape look like a parallelogram? How do you know it is a parallelogram?

What is its base? ____________________ What is its height? __________________

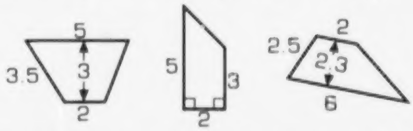
What is the area of the parallelogram? ____________________

How does the area of shape G compare with the area of the parallelogram? ____________________

What is the area of shape G? ____________________

Explain and justify your conclusion.

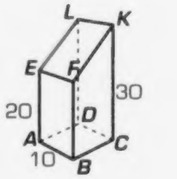
Does this method for finding the area of shape G work for other trapezoids?
Consider the trapezoids below. Find the area of each and explain why your method works.



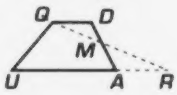
Now let's generalize. Call the lower base of the trapezoid *b*, the upper base *b′*, and the height *h*. How can we express the area of the trapezoid in terms of *b*, *b′*, and *h*?

Conjecture: The area of a trapezoid with bases *b* and *b′* and height *h* = ____________.

CHALLENGE PROBLEMS:



1. A new building has two exterior vertical walls in the shape of trapezoids, *BCKF* and *ADLE* (see diagram). There is a need to use a special vinyl covering on these walls for added protection. If the base of the building is a square 10 meters on a side and the other two vertical walls are rectangles with heights of 20 meters and 30 meters, how many square meters of covering will be needed to cover these walls?



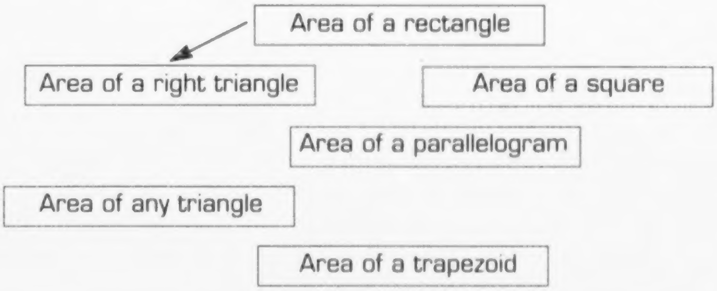
2. A student discovered another way of finding the area of a trapezoid *QUAD* (see diagram) by finding the midpoint *M* of one leg ($\overline{AD}$) and rotating triangle *QMD* around *M* so *Q* ends up at R. The student claims he has transformed the trapezoid into a triangle *QUR*, claims it's easy to find the area of the triangle, and finds it to be the same as the conjecture above. Do you think the student is correct? Explain.
3. A student explored finding the length of the midline of a trapezoid (the line segment joining the midpoints of the two legs of a trapezoid) and then discovered a new way to find the area of a trapezoid by using only the midline and the height. Can you discover what she did? Explain.

ACTIVITY 9D
FAMILY TREES

Every person has some ancestors. Who are some of your ancestors? How would you begin to create your own family tree?

Mathematical statements also have ancestors, namely, other mathematical statements that assure the truth of the statement. For example: If you know 7 + 7 = 14, then you know 2 x 7 = 14, and so 7 + 7 = 14 might be called an ancestor of 2 x 7 = 14. Logical relations like this can be expressed visually in the form of a family tree like the one shown below, where the arrow means, "If you know this, then you know this."

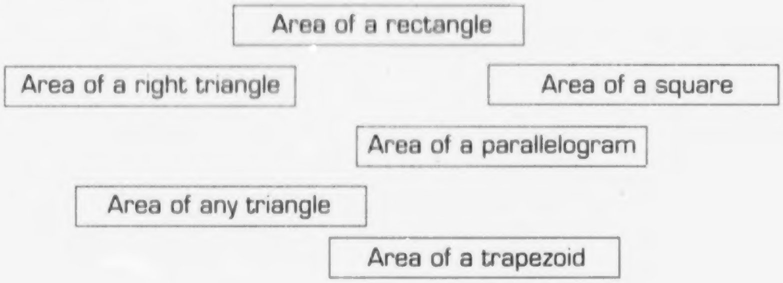
Let's consider building a "family tree" of the relationships for finding the areas of various shapes.


Area of a rectangle
Area of a right triangle
Area of a square
Area of a parallelogram
Area of any triangle
Area of a trapezoid

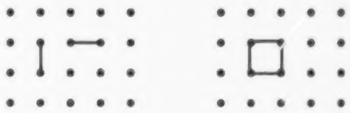
How can you explain or justify the arrow from area of rectangle to right triangle?

Now complete the family tree by drawing appropriate arrows to show a logical ordering of the formulas for area. Explain the meaning and placement of each arrow.

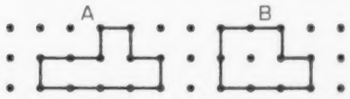
Recall that in the previous activities on finding the area of shapes, you found the area of each shape in several different ways. Complete the family tree below in a different way from the one above. For example, explain why someone might place an arrow from area of right triangle to area of any triangle. Now, as you place your arrows in the tree below in a different way, explain the meaning of each arrow and justify why it is correct.


Area of a rectangle
Area of a right triangle
Area of a square
Area of a parallelogram
Area of any triangle
Area of a trapezoid

ACTIVITY 10
GEOBOARD ACTIVITY—PERIMETER, AREA, AND PICK'S THEOREM

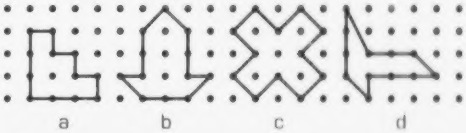


On a geoboard, the horizontal or vertical distance between two pegs represents one unit. The area of a square formed by joining four pegs with one-unit lengths represents one square unit.



1. On a geoboard, create the shapes shown at the left. If these were pens for animals, which would require the most fencing? The most floor covering? Explain.

2. If the area of □ is 1 square unit, what is the area of ◺ ? _______
If the area of □□ is 2 square units, what is the area of ◺ ? _______
Find the area of each of the following:



a. _______
b. _______
c. _______
d. _______

Boundary points of a geoboard figure are the points (pegs) that the rubber band touches; interior points are pegs inside the figure.

Figure *a* has 12 boundary (*B*) points and 1 interior (*I*) point.

Figure *b* has ____ boundary (*B*) points and _____ interior (*I*) points.

Figure *c* has ____ boundary (*B*) points and _____ interior (*I*) points.

Figure *d* has ____ boundary (*B*) points and _____ interior (*I*) points.

Below are examples of geoboard figures with different numbers of interior points. In the extra space provided for each case draw a figure of your own. Record the number of boundary (*B*) points and the area (*A*) of each figure in the chart at the right.

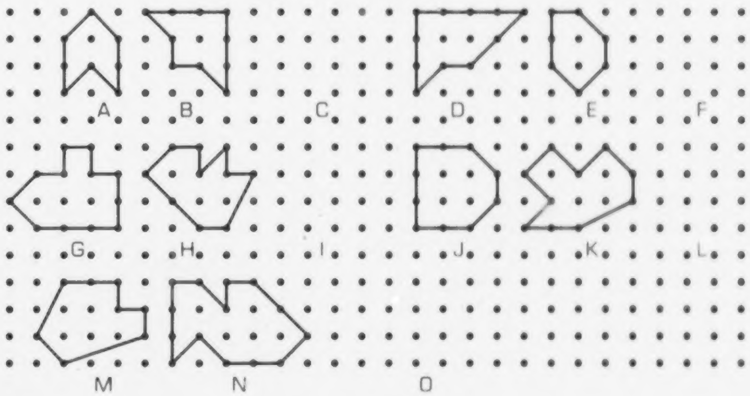


Figure	*I*	*B*	Area
A	1		
B	1		
C	1		
D	2		
E	2		
F	2		
G	3		
H	3		
I	3		
J	4		
K	4		
L	4		
M	5		
N	5		
O	5		

What patterns do you see? Describe them.

If you don't see a pattern, try drawing a few more figures.

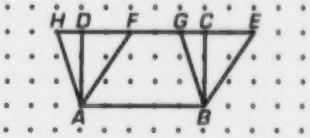
What do you think the area is if there are 8 interior points? ___ 10 interior points? ___

On the basis of the patterns you have discovered, what conjecture would you make for the area of a geoboard figure with *B* boundary points and *I* interior points?

Conjecture: Area of a geoboard figure = ________________________________.

ACTIVITY 10
(CONTINUED)

From your investigation you have discovered a famous theorem called **Pick's Theorem** about areas of geoboard figures. Does the pattern you discovered hold true for geoboard figures with zero interior points? Explain.



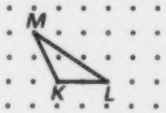
3. **Experiment 1.** Create a rectangle on your geoboard; call it *ABCD*.

What is the area of *ABCD?* _____With a second rubber band create a parallelogram on top of your rectangle so that the top base of the rectangle shifts two units to the right; call the figure *ABEF*. What is the area of *ABEF?*_____ Now with a third rubber band, create another parallelogram so that the top base of the rectangle shifts one unit to the left; call the figure *ABGH;* what is the area of *ABGH?*_____

What pattern do you notice concerning the areas of the three figures?

Explain why the pattern is true.

The method you have just used—namely, keeping the same base and shifting the top base (or points) in a direction parallel to the base and the same distance from the base—is called **"shearing."**

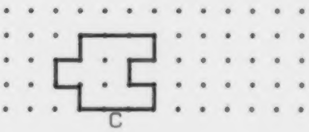


Experiment 2. Clear your geoboard and create a triangle on the geoboard; call the triangle *KLM*. Now apply the shearing process on the triangle by using several rubber bands to create three or four different triangles with the same base *KL*. Find the area of each triangle. What pattern do you notice concerning the areas of the triangles?
Explain why the pattern is true.

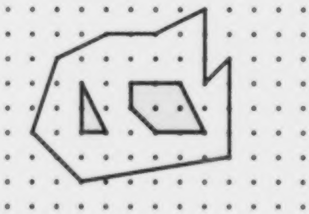
From the two experiments above, what conjecture(s) would you make concerning the shearing effect on rectangles and triangles?

Conjecture: ______________________________

CHALLENGE PROBLEMS:

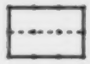
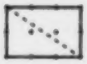
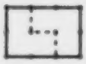


1. Create shape *C* on your geoboard. Perimeter = ____ ; Area = ____.
   Can you keep the *perimeter* of figure *C* the same and make the area smaller? Larger? Illustrate and explain.
   Can you keep the *area* of figure *C* the same and make the perimeter smaller? Larger? Illustrate and explain.

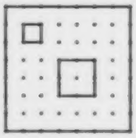


2. What if your geoboard figure has "holes" in it. (See diagram.)
   How would you find the area of this figure?
   Which are the boundary points when the figure has "holes"?
   Which are the interior points when the figure has "holes"?
   Investigate how to find the area of several geoboard figures with different numbers of holes. (Do not let the holes have any boundary points in common.) Does the conjecture you made in the previous activity still hold true? If not, how might you modify it?

3. On a geoboard, create a rectangle 3 units by 2 units. (*Caution:* Not 3 pegs by 2 pegs.)
   Using one rubber band, divide the rectangle into two equal regions. There are many different ways (at least 8) to do it; how many can you find? (*Note*: A way is not different if it is merely a reflection of another way; also remember the rectangle must be divided into only *two* equal regions with *one* rubber band.)
   Record them on dot paper. Here are a few:

ACTIVITY 11
SQUARES AND THE PYTHAGOREAN THEOREM



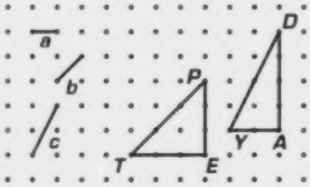
1. On your geoboard create squares with areas of 1, 4, 9, 2, and 5 square units. Record your results on dot paper and complete the chart below.

Area of square (in square units)	1	4	9	?	2	5
Length of side of square	1			4		
PeRIMeter of square	4					

Do you see any pattern? Describe it. ____

____

If the length of a side of a square is *s*, what is its area? ________ If the area of a square is *A*, what is the length of one side? ______________________________

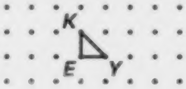


What is the length of each of the line segments shown at the left? *a* = _____ *b* = ______
c = ______

What is the peRIMeter of triangle *PET*? ____________

What is the peRIMeter of triangle *DAY*? ____________

2. In the center of your geoboard create right triangle *KEY*, with right angle at *E*, *KE* = 1 unit, and EY = 1 unit, as shown at the right.



With two rubber bands create a square on side *KE* and on side *EY*; now create with another rubber band a square on side *KY*. Use your L-square to check the right angles of your squares. Record your geoboard figure on dot paper. What is the area of the square on side *KE*? On side *EY*? On side *KY*? Explain how you found this area.

On your geoboard, let's explore other right triangles with different sizes of legs. Find the areas of the squares you build on the legs and on the hypotenuse. Record each result in the chart below.

Leg 1	Leg 2	Area of square on leg 1	Area of square on leg 2	Area of square on hypotenuse
1	1			
1	2			
2	2			
1	3			
3*	4*			

*Draw the triangle on dot paper unless you have a large geoboard.

Examine your chart. What pattern(s) do you see? Describe the pattern(s).

ACTIVITY 11
(CONTINUED)

If the lengths of the legs of a right triangle were called *a* and *b*, what conjecture would you make about the relationship of the areas of the squares on the two legs to the area of the square built on hypotenuse *c*?

Conjecture: In a right triangle with legs *a* and *b* and hypotenuse *c*, ______________________________

____.

Or symbolically you could write: ____

A famous Greek mathematician, Pythagoras, worked with right triangles and noticed a pattern. He described it this way:

> In a right triangle, the sum of the squares on the legs is equal to the square on the hypotenuse.

Did you discover the same pattern? Is it useful to know this pattern? Let's see.

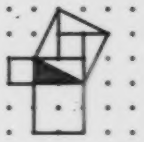
1. If a field is 14 m by 48 m, how many meters will you save if you run diagonally across the field instead of running around the two sides of the field? Explain.
2. Can a circular table top 3 m in diameter fit through a doorway with dimensions 1 m by 2.9 m? Explain.
3. A ladder 8 m long is leaning against the wall of a house. The foot of the ladder is 2 m from the wall. How high up on the wall does the ladder reach? Explain.
4. Is it possible to store a bamboo pole 3 m long in a closet 1 m × 2 m × 2.5 m? Explain.

CHALLENGE PROBLEMS:

1. If someone gave you the following set of directions, how many paces, measured in a straight line, are you away from your starting point? Explain.

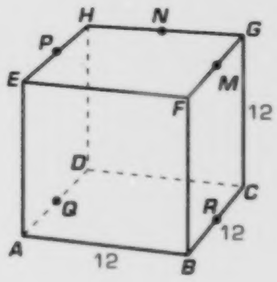
   Go forward 8 steps, continue to face the same direction and go left 7 steps, right 5 steps, left 12 steps, right 3 steps, left 4 steps.

2. In one of your geoboard constructions above, you constructed a square on the hypotenuse of a right triangle with legs 1 unit and 2 units and found the area of the square to be 5. Record that geoboard construction on dot paper and cut out the five pieces of the square on the hypotenuse. Now reassemble the five pieces so they fill the squares on the two legs of the right triangle. Is it possible? Why or why not?



3. Given cube *AG* with edge length = 12 cm. Points *M, N, P, Q*, and *R* are midpoints of edges *FG, HG, HE, AD*, and *BC*, respectively. Identify the geometric shape of each of the following cross sections of the cube:
   - a. *MPQR*
   - b. *EACG*
   - c. *BEG*
   - d. *MNDB*

   Can you find the area of each of the cross sections above? If yes, find each area; if no, explain why the area cannot be found.



ACTIVITY 12
MEASURING A CIRCLE

Object	*d*	*C*	*C*:*d*

Bike wheels, cans, and other round things (circles) come in many sizes. If you know the longest distance across (diameter) an object, can you predict the distance around (circumference) the outside of the object?

Example: If the diameter of a bike wheel is 70 cm, approximately how far does the bike move in one revolution of the wheel? In 10 revolutions?

Let's explore: Your group should carefully measure the diameter (d) and circumference (C) of five or six different objects (large and small) and record the data in the chart at the left. In the last column find the ratio of $C{:}d$ to two decimal places. Use your calculator.

Do you notice any pattern(s)? Describe the pattern(s). Check with another group; did they find a similar pattern?

Did you find that the ratio $C{:}d$ is a little more than 3, say, 3+? So what is your conjecture?

Conjecture: Circumference of a circle = ____.

Try measuring the diameter of a circle with a piece of string and then find how many times the length of the string (diameter) fits around the circle. Does your result agree with your conjecture? Isn't it amazing that everyone around the world who measures the diameter of a circle will find that it fits around the outside of the circle a little more than 3 times!

Let's apply the result of this exploration to the bike problem above. The diameter = 70 cm. How far does the bike wheel travel in 1 revolution? What is finding the distance traveled in one revolution the same as?

Let's consider another problem:

The local pizza shop has a special offer: Two 15-cm pizzas for $6
One 30-cm pizza for $5

If you really like the crust of the pizzas that this shop makes, which is the better deal? Why?

If you are not so fond of the crust but really like the rest of the pizza that the shop makes, can you find out which would be the better deal and by how much? But how do you find how big the pizza really is? How do you find its area? Let's explore.

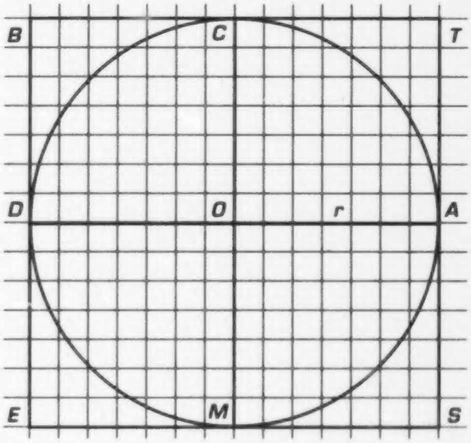
Exploration: How to find the area of a circle?

On centimeter grid paper, draw a circle with radius = 7 cm. Draw a square around the circle so that each side of the square touches the circle (see the diagram at the right). Draw the perpendicular diameters so that they divide the circle into four equal parts (sectors). They also divide square *BEST* into four equal squares.

What is the area of square *BEST*? Of square *COAT*?

To generalize, let's call the radius of the circle r, that is, $OA = r$.

In terms of r, what is the area of square *BEST*? Of square *COAT*? Of square *BDOC*? Of square *DOME*? Of square *MOAS*?



ACTIVITY 12
(CONTINUED)

Is the area of the circle larger or smaller than square *BEST*? Why?

Let's try to squeeze the area of the sector *COA* of the circle into the three corners *B*, *E*, and *S* to fill up the small squares. To do this, check off 1 square unit in the sector and 1 square unit in one of the corners *B*, *E*, or *S*. Continue doing this and then check off parts of boxes in the circle that match comparable parts of boxes in the three corners until all the spaces in the three corners are completely filled with checks. (*Caution:* Do not check off any boxes in corner *T*!) Now look at your diagram: Have you almost squeezed all of sector *COA* into the three corners?

What does that mean about the area of the whole circle in relation to the three squares *BDOC*, *DOME*, and *MOAS*? Describe.

How does the area of the whole circle with radius *r* compare with the areas of the three squares with side equal to *r*? What conjecture would you make?

Conjecture: The area of a circle with radius *r* is ______________________________

____.

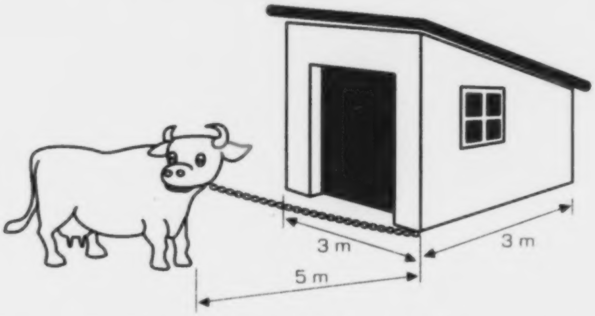
Symbolically you could say area of a circle of radius *r* = ______________________.

(*Note:* Some people like to replace 3+ with a value like 3.14 or 3.1416...; others have assigned a Greek letter π (pi) to the value 3+. So you sometimes see formulas like these: Circumference of a circle = πd or area of a circle = πr^2. But remember that π stands for a number—namely, 3+.)

Let's return to the pizza problem: Can you now decide which is the better deal in terms of the amount of pizza you get in two 15-cm pizzas compared to one 30-cm pizza for the given prices?

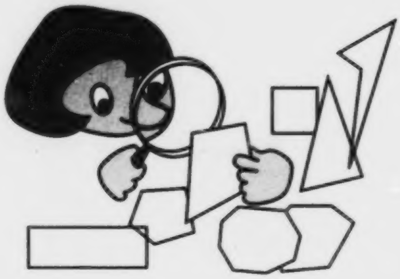
CHALLENGE PROBLEMS:

1. A student claims to have discovered another way of finding the area of a circle by cutting a circle into 8 equal pie-shaped pieces and rearranging the pieces into a shape resembling a parallelogram. Then the student said the area of the circle was equal to the area of the parallelogram and found the same result as conjectured above. Try this same experiment. Did you get the same result? Is the student correct? Why or why not?
2. The broadcasting radius of a local televison station WSEE is 50 km. In how many square meters does the viewing audience of WSEE live? If you were to choose a location for a television station in a city, what questions would you investigate before making a decision for the location?
3. A farmer tied his cow, Mooey, to a post with a rope 5 m long. How many square meters of grazing ground does Mooey have? If the farmer moves Mooey to another field and ties him to a post with the same rope at the corner of a closed shed 3 m by 3 m in the field (see diagram), how many square meters of grazing ground does Mooey have?



ACTIVITY 13

INVESTIGATION: PERIMETERS AND AREAS OF SIMILAR TRIANGLES

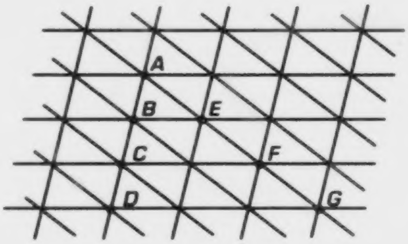


A newspaper reports that a 13-year-old, HiTech Tesha, has investigated a Polygon family and discovered some startling facts about similar Polygons. She will not reveal her findings but challenges others to begin by investigating two basic questions:

1. What relationships exist in similar triangles between corresponding sides and the perimeters of the triangles?
2. What relationships exist in similar triangles between corresponding sides and the areas of the triangles?

Respond to HiTech Tesha's challenge by doing your own investigation using computer software such as the Geometry Supposer or Euclid's Toolbox.

But *first*, you may want to do some visual thinking about the problem and explore some relationships among similar triangles. For example, look at the triangles in this grid made of sets of parallel lines.



Name some similar triangles.

Look at triangle *ABE* and triangle *ADG*. Are they similar? Why or why not?

What is the ratio of their corresponding sides? Their perimeters? Their areas?

Or look at triangles *ABE* and *ACF* and answer the same questions.

Or look at triangles *ACF* and *ADG* and answer the same questions.

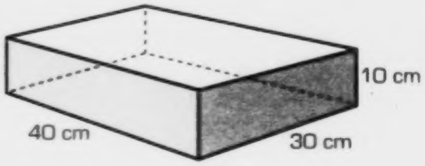
What hypotheses might you make about similar triangles from these preliminary examples?

And now to the *challenge*, using your own computer software:

What data will you need to collect to do this investigation?
How will you set up your database?
What different types of triangles should you consider?
For example: acute, right, obtuse?
isosceles, equilateral, scalene?
What ratios will you explore?
How will you record your data?

Finally, prepare a report on your investigation. The report should include the data collected, your analysis of the data, your conclusions—in particular, the conjectures you would make that are supported by the results of your investigation. Be sure you justify all your conclusions. Also, indicate how you might extend your investigation—that is, what other questions might you investigate in relation to similar figures?

ACTIVITY 14
SURFACE AREA



Problem: A gift you bought for a friend is in a rectangular box 40 cm × 30 cm × 10 cm. You decide to cover the sides of the box with gold foil to make it look attractive. How much gold foil will you need to cover this gift box?

What ideas did your group consider in exploring this problem? Describe how you would find the surface area of the box (also called a rectangular prism).

The surface area of a prism is the sum of the areas of each face and base of the prism.
What is the sum of the areas of the faces of your gift box?
What is the sum of the areas of the bases of your gift box?
What is the surface area of your gift box?

Let's generalize to other similar gift boxes. If the dimensions of a rectangular prism are *l*, *w*, and *h* where *l* = length, *w*, = width, and *h* = height, state your conjecture for finding its total surface area.

Conjecture: The surface area of a rectangular prism = ________________________________.
Justify your conjecture.

TENNIS BALLS

If your gift were a can of tennis balls (the shape of a right circular cylinder), the surface area would be the area of the rounded surface of the cylinder (called the lateral area) and the areas of the two bases.

Let's experiment to see how to find the area of this rounded surface (the lateral area).

Wrap a piece of paper around the can; take it off, spread it out flat, and describe the shape of the paper. Does it look like the "label" off the can? How can you find the dimensions of the label?
What relation does the length of the label have to the can? Describe.
What relation does the width of the label have to the can? Describe.

What is your conjecture about finding the lateral area of the can (right circular cylinder) with radius = *r* and height = *h*?

Conjecture: The lateral area of a right circular cylinder = ________________________________.
Justify your conjecture.

If you made a paper model of a can (a right circular cylinder) and flattened it out, what would it look like? Sketch a picture of this flattened-out paper model, called the "net" of the can.

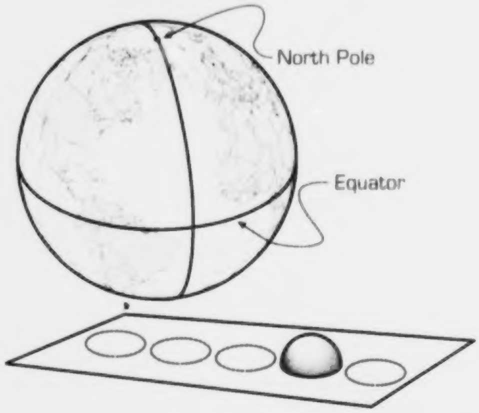
From this visual model, what conjecture would you make concerning the total surface area of a right circular cylinder with radius *r* and height *h*?

Conjecture: The total surface area of a right circular cylinder = ________________________________.
Explain why your conjecture is true.

Let's apply your conjecture.

What would be the cost of painting the exteriors of five large storage tanks that are 10 m high and 6 m in diameter. Each liter of protective exterior paint costs $8 and covers 40 square meters. (The bottoms of the tanks are not to be painted.)

ACTIVITY 14
(CONTINUED)



Problem: How would your group find the surface area of a sphere, such as a globe, an orange, a ball?

On a globe, the equator and longitudinal circles are "great circles" of the sphere. For example, if you slice an orange, any slice formed by the knife's going through the center of the orange is a great circle.

Experiment: Take an orange and cut it in half. Take one of the halves and put it on waxed paper; trace around the bottom several times to form four or five circles on the waxed paper. Now carefully peel the orange (both halves) and place the small pieces of flattened-out peel in the circles on the waxed paper and see how many circles you can fill with the total peel of the orange.

How many circles did your group fill? How many circles did other groups fill? What is your conjecture about the surface area of the orange in terms of the number of its great circles covered by the peel?

Conjecture: The surface area of the orange = the area of ______ great circles of the orange.

Measure the radius of your orange: radius = ______________________________.

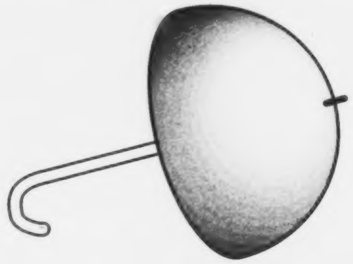
What is the area of one of your great circles of the orange? ______________________________

All of the orange peel represents the total surface area of the orange.
So what is the surface area of the orange? ______________________________

If the radius of a sphere is r, make a conjecture concerning the measurement of the surface area of the sphere in terms of r.

Conjecture: The surface area of a sphere with radius r = ______________________________.
Justify your conclusion.

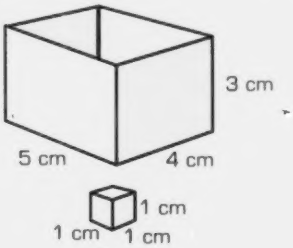
Application: An umbrella designer has created a new model for an umbrella that when opened, has the form of a hemisphere with a diameter of 1 m. If a dozen sample models are to be made using a special waterproof material, approximately how much fabric will be needed, allowing 0.5 m for seams and waste for each model? Explain your plan, strategies, and obstacles and how you solved the problem.



CHALLENGE PROBLEMS:

1. Examine the conjectures you made in the activity above. If you were to consider placing these conjectures in the family tree you created in Activity 9*d*, where would you place them? Explain the relationships in order to justify your placements.
2. Two cylindrical lampshades that are 40 cm in diameter and 40 cm high are to be covered with new fabric. The fabric chosen comes 1 m wide. If you decide to buy 1.5 m of this fabric, will you have enough material to complete the task? Justify your answer.
3. Distances between two points on a sphere are always measured by the length of the arc of the great circle passing between the two points. The length of the equator (a great circle) of the earth = 40 000 km. Your task is to use a piece of string and a globe of the world to find the shortest distance between Hong Kong and New York City. Explain how you analyzed the task and how you solved the problem.

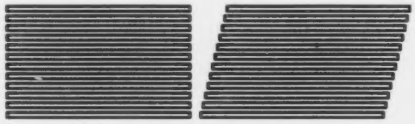
ACTIVITY 15
VOLUME



Problem 1: If you wanted to fill the box (rectangular prism) at the right with centimeter cubes, how many would you need?

If you packed it neatly, how many would be on the bottom layer? How many such layers would there be in this box?

Is there a quick way to find how many cubes the box will hold? Explain.



Exploration: If in a lumber yard you see a stack of plywood sheets stacked straight up and nearby you see another slanted stack of the same number and same size of plywood sheets, which stack has the greater amount of wood in terms of cubic meters? Explain.

Try it with a stack of index cards arranged in straight stacks, slanted stacks, and spiral stacks, each stack having the same number and size of cards. Which stack has the greatest volume? Explain.

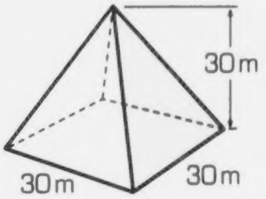
Try this with stacks of crackers of different shapes (e.g., triangle, square, hexagon, circle). These stacks would represent a triangular prism, a square prism, a hexagonal prism, and a circular cylinder. If you have a straight stack of crackers or a slanted stack of the same crackers, both stacks having the same height, which stack has the most volume? How would you find the area of the bottom layer? How would you find the volume?

From the exploration above, what conjecture would you make about the volume of a prism or a circular cylinder in relation to the area of its base (b), that is, the area of the bottom layer, and its height (h), that is, the number of layers?

Conjecture: Volume of a prism (or cylinder) with area of base (b) and height (h) = ________________.

Let's apply your conjecture:

1. Which will carry the most water, two pipes, one with a 3-dm radius and the other with a 4-dm radius, or one pipe with a 5-dm radius? Explain.
2. A company delivers 36 cartons of paper to your school. Each carton measures 40 cm × 30 cm × 25 cm. Is it possible to fit all the cartons in a storage closet 1 m × 1 m × 2 m? Explain and justify your conclusion. Try justifying your conclusion with a visual explanation.



Scale: 1 cm : 15 m

Problem 2. You have studied about the pyramids in Egypt. You want to make a scale model of a pyramid (see figure at the left) for a display using modeling clay. How much modeling clay will you need?

Discuss with your group how you might investigate this problem. What strategies will be useful in this exploration?

Is the volume of a pyramid related to the volume of a prism in any special way if they both have equal bases and heights? How can you find out?

Examine your models.

ACTIVITY 15
(CONTINUED)

Experiment: Use your models of a prism and a pyramid that have equal bases and equal heights. Estimate how their volumes compare. What fractional part of the prism will be filled by the contents of the pyramid? Now check your estimate by having one member of the group fill the pyramid with water and pour it into the prism. Do you want to revise your estimate? Continue pouring as before until the prism is full. What did you discover? How many "pyramidfuls" did you need to fill the prism?

Try the same experiment using a cone and a cylinder with equal bases and equal heights. Estimate how their volumes compare and then check your estimates. How many "conefuls" are needed to fill the cylinder? Was your estimate close?

From these two experiments with pyramids and cones, what conjecture(s) would you make concerning the volumes of these shapes in relation to a prism and a cylinder?

Using the symbols *b* (area of base of prism/pyramid) and *h* (height of prism/pyramid), express your conjecture for the volume of a pyramid.

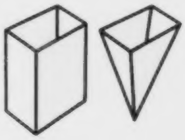
Conjecture: The volume of a pyramid = ____.
Can you now solve the original problem? How much modeling clay will you need for creating your model?

Similarly, write a conjecture for finding the volume of a cone in terms of the area of its base (*B*) and its height (*H*).

Conjecture: The volume of a cone = ____.
Justify why your conjecture is true.

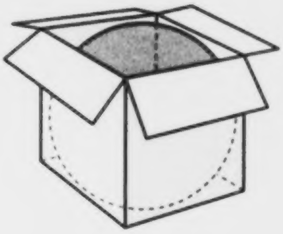
Let's apply your conjectures:

1. A theater decides to change the shape of its popcorn container from a rectangular box to a pyramidal box (the tops of both boxes are the same size and both containers are the same height) and charge only half as much. Is this a bargain? Why or why not?



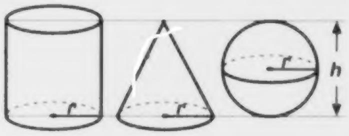
2. A glass in the shape of a cone has a diameter of 8 cm. If the glass contains 200 mL (or 200 cc), what is the greatest depth of the liquid? Explain.

Problem 3. A manufacturer of globes (approximately 1 m in diameter) packs the globes in one cubic meter boxes for shipping. How much protective packaging material (e.g., Styrofoam peanuts) is needed for a shipment of 100 globes if the Styrofoam is inserted in each box to insure safe shipment?



Discuss with your group how you might investigate this problem. What strategies will be useful in this exploration? What information is needed? How can you find the volume of a sphere?

Experiment: Consider using your models of a cylinder, cone, and sphere that have equal diameters and equal heights. Notice that if the radius of the sphere is *r*, the height of the sphere in terms of *r* is _____. Also in terms of *r*, the height of the cone is ____, and the height of the cylinder is ____.



ACTIVITY 15
(CONTINUED)

The following experiment involves the principle of displacement. If you drop a big rock in a container full of water, what happens to the water? What relationship exists between the volume of water that overflows and the rock? Explain.

Let's try using this displacement principle to find the volume of the sphere. Use a cylinder and a sphere that have the same radius and the same height. Fill the cylinder with water and gently place the sphere in the water. (*Caution!* Be sure to place a shallow container under the cylinder to catch the water displaced.)

How will you measure the displaced water with your models?

Try pouring it into the cone. How many "conefuls" of displaced water are there?
One "coneful" is what part of the volume of the cylinder? ____
Two "conefuls" is what part of the volume of the cylinder? ____

So the volume of the sphere is what fractional part of the volume of the cylinder?____
How can you express the volume of this cylinder in terms of its radius r?
What is the area of its base? _______ What is its height? _______
Hence, the volume of this cylinder in terms of r = _____________ .

Since you discovered by displacement that the volume of the sphere was a fractional part of the volume of the cylinder, what is your conjecture concerning the volume of the sphere in terms of r?

Conjecture: The volume of a sphere with radius r = ________________.

Using this conjecture, can you now solve the globe manufacturer's problem? How much protective packaging material will be needed? _______ This manufacturer estimated that about 1/2 m^3 of Styrofoam peanuts is needed for each box shipped. Was this a good estimate? Explain and justify your response.

CHALLENGE PROBLEMS:

1. Which is the better buy, oranges 7 cm in diameter that cost 20 cents each or oranges 9 cm in diameter that cost 30 cents each? Explain.
2. A cylindrical glass 7 cm in diameter and 10 cm tall is filled with water to a height of 8 cm. If a ball 5 cm in diameter is dropped into the glass and sinks to the bottom, will the water in the glass overflow? Explain and justify your response. What if the ball floats instead of sinks, will the water overflow? Explain.
3. The Licking Good Ice Cream Shop is about to advertise giant spherical scoops of ice cream 8 cm in diameter! The owner of the shop wants to be sure there is enough ice cream and wonders how many scoops can be obtained from each cylindrical container of ice cream. The containers are 20 cm in diameter and 26 cm tall. How can the owner determine how many containers of ice cream to order?
4. A doughnut shop decides to make minidoughnuts in addition to its regular doughnuts. The regular doughnuts have a diameter of 8 cm with a 2-cm hole, and the minidoughnuts have a 6-cm diameter with a 1.5-cm hole. The price of a regular doughnut is 40 cents, and the shop decides to sell the minidoughnuts for 20 cents each. Is this a bargain for the customer? Explain. Will this price make money for the owner? Explain.
5. **Experiment:** Take two sheets of paper about the same size as this activity sheet; roll one sheet vertically and the other sheet horizontally and tape each to form a right circular cylinder. If you were to fill each with popcorn, would they contain the same amount of popcorn? Explain and justify your findings.

CLUSTER C
MAKING CONNECTIONS THROUGH INTERDISCIPLINARY MEASUREMENT ACTIVITIES

Activity 16: Planet Jumping

Activity 17: Paintbrush Measure

Activity 18: Scale Models
  a. Interpreting a Floor Plan
  b. Building a Model of an Apartment to Scale

Activity 19: Field Mathematics—Making and Using a Hypsometer

Activity 20: How Change in Size Affects Measurements
  a. Measuring Shadows
  b. Grid Enlargements and Building Staircases

Activity 21: How Change in Scale Affects Measurements
  a. "Lost and Found"
  b. *Honey, I Shrunk the Kids*

Activity 22: Size and Form of Buildings—Architecture
  a. The Shape of Things
  b. Be an Architect
  c. Building with Paper

Activity 23: Size and Form of Animals—Biology
  a. Heartbeats
  b. The Weight of Wetness

Activity 24: Density
  a. Floating Fruits
  b. Fresh vs. Salt Water and Population

OBJECTIVES FOR CLUSTER C

1. To provide activities that give students insight into how mathematical thinking is an integral part of other disciplines
2. To have students engage in activities that show how mathematics and measurement are useful tools in many areas of learning
3. To have students value mathematics and their ability to apply mathematical thinking and modeling to solve problems that arise in other disciplines and in the real world

TEACHING NOTES FOR CLUSTER C

The activities in this cluster illustrate how measurement skills can be applied with a purpose, finding patterns that have important implications in other curriculum areas, in this instance, architecture and biology. Interdisciplinary studies serve many functions, suggesting to students the interconnectedness of learning and, in particular, how mathematics can add insight and be a useful tool in many areas of life. The initial activities establish how change in scale affects measurements of length (height, width, perimeter, etc.), surface area, volume, and weight. Further activities explore implications of the "scale factor" for the design of buildings and for biology.

Activity 16. Planet Jumping

Materials: Copy of Activity Sheet 16 for each student; metersticks, 20-cm rulers (see Appendix)

This activity involves calculating the distance a person could jump on planets that have different gravitational forces. Newton's law of gravitation states that two bodies are attracted to each other with a force directly proportional to the product of their masses and inversely as the square of the distance between them. It is important to stress that the law of gravity is based on a planet's mass, not on its size. Therefore, although Mars is much larger than Mercury, their masses are the same and a person would weigh the same amount on both planets (see the "Elvis" chart on Activity Sheet 16).

Time should be spent discussing how "mass" differs from "weight." Mass refers to the amount of "stuff" inside a body, whereas weight refers to the gravitational force that acts on that body. The mass of an object never changes regardless of where it is, because the amount of stuff inside does not change. However, the weight of something depends on the gravity pulling on it. Since the moon is only 1/6 as massive as the earth, the weight of an object on the moon would be 1/6 of that on the earth. At the same time, a person would be able to jump 6 times farther on the moon. The weight of something can even vary on Earth: the closer something is to the center of the earth, the stronger the gravitational pull, whereas the farther away an object is from the center of the earth, the weaker the gravitational pull. However, it should be stressed that the amount of weight change from the highest point on Earth (Mount Everest) to the lowest point (the Dead Sea) is not significant (a few ounces on the average human).

The main background necessary to complete this activity is experience in multiplying fractions and some work with ratios and proportions. As students are calculating their distances, they should estimate their answers first. For example, if they were jumping on the moon, they would know their distances would be 6 times farther, whereas on Neptune their answers should be only a little less than their distances on Earth. Students could round off Jupiter's gravitational pull from 5/12 to 1/2, so the estimate of their distances on Jupiter should be only a little less than half that on Earth.

At the same time students are calculating their jumps, they should also be comparing Earth's gravity with that of other planets. What should emerge is an inverse relationship: the greater the mass of a planet, the shorter the distance one is able to jump. At the same time, a direct relationship emerges: the greater the mass of the planet, the stronger its gravitational pull.

Extensions: The "Elvis" chart is meant to show the gravitational pull relative to each planet so that students can make inferences about the mass of each planet in comparison to that of the others. Again, the direct relationship is reinforced as students see that an increase in the mass of the planet results in an increase in the weight of an object (Elvis Presley).

The final section deals with the path of a ball traveling on Earth in comparison to other planets. Students can use the information from the activity to determine the distance a ball would travel on other planets and estimate those distances where fractions are not specifically provided. The activity could be further extended in several other ways. For example, the students could research the mass of each planet and plot the

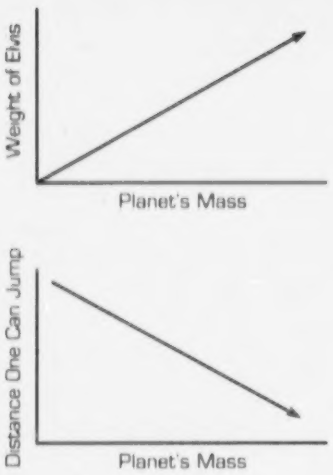


Fig. 16a

planet's mass versus the weight of Elvis on that planet or graph the planet's mass versus the distance one could jump to form two different statistical relationships (see fig. 16a).

It would also be possible to extend the activity into the use of reciprocals. For example, one can jump 6 times farther on the moon; therefore, the moon has 1/6 of Earth's gravity. Since one can jump 5/6 as high on Neptune as on Earth, we would describe Neptune's gravitational field as being 6/5 (or 1 1/5) times stronger than Earth's. Using ratios and proportions, it would be possible to determine the gravitational pull of all planets listed on the "Elvis" chart. For example, since Elvis would weigh 125 kg on Saturn as compared to 116 kg on Earth, we would know that Saturn has a stronger gravitational pull. We can thus form the ratio 125/116, indicating that Saturn's gravitational pull is 1.1 times stronger than Earth's. If we turn this into the fraction 11/10, we would find that a person could jump only 10/11 as high on Saturn as on Earth.

The use of percents could also be brought into this activity by comparing the weight of Elvis on other planets to his weight on Earth. For example, by taking Elvis's weight on Pluto (6 kg) and dividing it by his weight on Earth (115 kg), we would arrive at the conclusion that Pluto has a gravitational pull that is about 5 percent that of Earth's. By turning 5 percent into a fraction and finding its reciprocal (100/5, or 20), we would find that Earth is 20 times more massive than Pluto.

	d (dist)	6*d*	(5/6)*d*
A	B Earth	C Moon	D Neptune
Long jump	1.2 m	7.2 m	1.0 m
Vertical jump	0.6 m	3.6 m	0.5 m

Fig. 16b

In this activity students can use a spreadsheet to great advantage to investigate and predict. For example, they can list the different physical activities they want to investigate (vertical jump, long jump, ...) in one column (for example, column A) of the spreadsheet and the records set on Earth in another column, say, column B (see fig. 16b). Then to see what the records would be on the moon, simply set the spreadsheet to make every cell in column C equal to 6 times the value of the corresponding cell in column B. Automatically students will get the corresponding records. To get the records on Neptune, set column D equal to 5/6 the value in column B. Students can now ask what the gravity of a planet has to be, compared to that of Earth, for a particular student in the class to be able to long jump the length of a football field. By trying different values in the spreadsheet, they can zero in on the answer or calculate it theoretically and then put that value into the spreadsheet to verify their answer. Of course this problem with multiplication by 6 or 5/6 on a variety of numbers is also a perfect opportunity for using the Explorer (fraction) calculator with the constant multiplication and division features. See also the discussion of the use of a spreadsheet in Teaching Notes for Activities 6, 13, and 23*a*.

Activity Sheet 17. Paintbrush Measure

Materials: Copy of Activity Sheet 17 for each student; collection of nonstandard measuring units such as stacks of index cards (3 × 5, 4 × 6, 5 × 8); sheets of paper from a small pad; set of congruent tiles (or stamps); graph paper; paper models of a square centimeter, square decimeter, and square meter; 12 small sippers/stirrers

An activity-oriented approach, similar to that used for length, should be used to develop the concept of *area* as *covering*. Before doing this activity, students should discuss what kind of a unit they might use to cover (i.e., to measure the area of) selected classroom items (e.g., the top of their desks, cover of their textbook, surface of a bulletin board). Have students measure classroom items with different nonstandard units. A discussion of the results should suggest the need for a standard unit for covering a region or measuring the area of a region (e.g., square centimeter, square decimeter, square meter).

◆ ◆ ◆ ◆ ◆ ◆ ◆ ◆

Students should complete the Paintbrush activities noting that the standard unit of measure in this activity is a square centimeter. To demonstrate on an overhead projector the brush stroke effect, place twelve small sippers together to form a rectangle; then use a pencil or straw to gently push the sippers to form a parallelogram without right angles; finally use a circular object (top of a jar) to gently push the sippers to form a rounded brush stroke (see fig. 17), thus showing that the original area does not change. Students usually enjoy this demonstration. The challenge problems in this activity involve square decimeters and square meters.

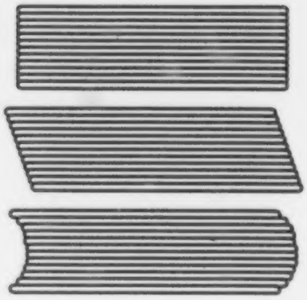


Fig. 17

Answers to Challenge Problems:

1. The quilt is actually 21 dm by 18 dm and so contains 378 patches with 42 patches of each color. The amount of fringe needed is 7.8 m.
2. The floor actually measures 120 dm by 50 dm. Hence 6000 tiles are needed and will cost $120.

Activity 18a. Interpreting a Floor Plan

Materials: Copy of Activity Sheet 18*a* for each student, centimeter grid paper, centimeter rulers (see Appendix)

Working in groups of five or six, each student is to write down everything he or she knows about the apartment shown in the floor plan. One person in each group should then report the group's findings to the class.

Activity 18b. Building a Model of an Apartment to Scale

Materials: Copy of Activity Sheet 18*b* for each student, centimeter rulers, toothpicks, straws, tape, pipe cleaners, contact paper

Students should have some knowledge of ratio and proportion before doing this activity. Each group of students will get a different floor plan. The activity gives the students hands-on concrete experiences in constructing a model to scale. Working in cooperative groups, students should complete the work outlined on Activity Sheet 18*b*. As the teacher circulates among the groups, questions should be raised and alternative strategies might be posed for consideration. Explanation and justification of students' work should be sought. Groups should share and compare observations made in their investigations.

Assessment—project: Constructing your own scale model: Before engaging in this activity, students will need prior experience with tasks similar to those in Activity 18*a* and 18*b*. Students select their own materials for this activity. In their cooperative learning groups, students choose their own room, whether it is a bedroom, living room, kitchen, bathroom, ... Students make a scale drawing of their room and its furnishings after taking appropriate measurements; they then select a scale from which to build a model of this room. One seventh grader decided to make a scale model of the three tallest buildings in the United States (see fig. 18). After constructing their scale model, students write up a summary of all their work, including learning experiences they encountered, obstacles they had to overcome, and whether they enjoyed or did not enjoy the project with reasons why.

Fig. 18

Activity 19. Field Mathematics—Making and Using an Hypsometer

Materials: Copy of Activity Sheet 19 for each student, cardboard sheets (for backing), centimeter grids, straws, string, weights (old keys, washers), tape

BEST COPY AVAILABLE

◆ ◆ ◆ ◆ ◆ ◆ ◆ ◆

Some knowledge of ratio and proportion and similar triangles is needed in this activity. Allow some time for class discussion of the intial problem on finding the height of the tree. The discussion might include finding the height of the tree by shadow reckoning as described in the problem in the Introduction of this book on page 4 (see fig. 4). In introducing the use of the hypsometer in the initial problem, students should see the need for measuring the distance from the trunk of the tree to where they are standing, notice that the hypsometer has four right angles, and recognize that there are two similar triangles, namely, triangle *ABC* and triangle *DEF*. (They are similar because right angle $C \cong$ right angle F, and angle $BAC \cong$ angle EDF, since they are both complements of angle *ADE*.) This means that $AC{:}DF = BC{:}EF$, since corresponding sides of similar triangles are in proportion. Given this information, the height of the tree can be found using the readings from the hypsometer.

Each group of students should make their own hypsometer. They will need some practice in learning how to read data from the hypsometer as they are using it. Each group should plan and implement a project for using their hypsometer to measure the heights of buildings, trees, and poles in the local area. Each group leader should assign specific tasks to group members so that the data are recorded accurately and checked. The whole class should engage in a discussion of the plans and implementation of each group project, including a consideration of future activities using a hypsometer in other real-world settings.

Activity 20a, b. Measuring Shadows; Grid Enlargements and Building Staircases

Materials: Copy of Activity Sheets 20*a* and 20*b*, a few common objects (file card, bottle cap, spoon, key, cut-out triangle), flashlight, piece of cardboard, overhead projector, transparent centimeter grid, centimeter ruler, large sheet of paper, collection of centicubes or cube-o-grams, protractor, calculator

Prerequisites for these activities are the concepts of ratio, enlargement, and shrinking and an ability to measure length, height, perimeter, surface area, and volume. Set up the projector and secure a large sheet of grid paper to the wall. Students should study whether the projected image is in fact similar to the image on the transparency by measuring the angles. They should discover that the light rays from the center of the projector must be perpendicular to the wall. Then they can experiment with finding and measuring the lengths and area of different sizes of images by sliding the projector closer to, or farther away from, the wall.

After completing Activity Sheets 20*a* and 20*b*, students might be challenged to find similar cans or boxes and measure the weight of water contained in them. (For example, they might use cardboard quart and half-gallon milk containers with the quart container cut down to form a container similar to the half-gallon one; see fig. 19.) In exploring this challenge, students should make conjectures and then consider whether their actual findings are consistent with their findings in Activity 20*b*.

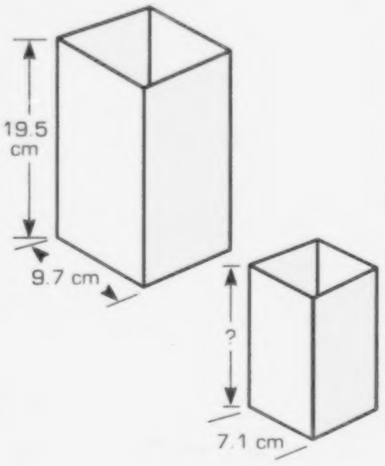


Fig. 19

Note: It is possible to separate Activity 20*b* into two pages. Combine the top activity with the idea of measuring the feet of different sized people (see Activity 5). The bottom activity could be expanded to consider the volume of similar solids (see Activity 15).

Reference: The curriculum unit *Mouse and Elephant: Measuring Growth* (see Shroyer and Fitzgerald [1986]) contains a very detailed sequence of activities involving the effect of enlargement on measurements of length, surface area, and volume.

Activity 21a. "Lost and Found"

Materials: The cards on Activity Sheet 21*a* (each to be cut out and placed in separate bags with an object as described), objects for bags

Students can be motivated for this activity by any of the classics of children's literature related to change in scale, such as *Stuart Little, The Borrowers, Alice in Wonderland,* or *Gulliver's Travels.* Discussion of methods of finding answers can lead to understanding how change in scale affects measurements of diameter, height, area, and capacity. For example, students might reason that four of the napkins cover a card table, so the person using the napkin as a card table cover would be 1/4 the height of a normal person. But in fact when length and width are halved, the area is quartered, and so the person should be 1/2 the height of a normal person. Similarly, students might find that eight of the doll's teacups fill one normal teacup, but this does not mean that the person is 1/8 the height of a normal person, but 1/2.

Assessment: Make up a "Lost and Found" game to challenge your friends with different real-world articles or objects that you have collected.

Activity 21b. Honey, I Shrunk the Kids

Materials: Copy of Activity Sheet 21*b* for each student, centimeter rulers, some Cheerios

The popular films *Honey, I Shrunk the Kids* and *Honey, I Blew Up the Kids* furnish another motivation for the consideration of scaling. In the first film, four children get shrunk to such a small size that traversing the lawn in front of their house becomes a huge challenge in a world of menacing giant insects and huge plants. Activity Sheet 21*b* shows the illustrations on the box in which the home video is marketed; these illustrations can supply some interesting challenges. The sheet can be used as a transparency for an overhead projector or can be distributed to students. In one picture, one of the children appears to be about 1/2 the length of the normal-sized father's nose. In another picture, a child is hanging on to a Cheerio floating in a bowl of milk, and it appears that the arm length of the child is about 1/6 of the diameter of the Cheerio. Ask students if these scales are consistent; also ask if the pictures agree with the publicity statement that the children are shrunk to 1/4 inch tall. Perhaps students will end up trying to measure the length of an adult's nose in Cheerios. Keep the activity open-ended.

Assessment—journal entry: Write a letter to the designer of the artwork describing your investigation and noting your findings and inconsistencies (if any) in the artwork *or* write a short story to tell how you would feel about being very small or very large (as in *Honey, I Blew Up the Kids*).

Activity 22a. The Shape of Things

Materials: Copy of Activity Sheet 22*a* for each student, class set of calculators

To complete this activity students need to have some background similar to that supplied in the activities in Cluster B where generalizations were developed for finding surface area and volumes of geometric figures. Students will find this activity interesting but quite challenging. Students should check their estimates for the cube root of a number by using their calculators.

The size of a building is often limited by the properties of materials available for construction. One may be able to build a structure of a certain size out of a material—say, wood—but not be able to build a similar, larger structure of the same material. As an example, imagine placing a wooden meterstick between two tables to span a gap of a little less than one meter. The stick will be able to carry a load much greater than its own weight. But if you constructed a wooden beam that was 100 times as big in each direction to span a gap of nearly 100 meters, it would sag and break under its own weight. This effect of scale on strength is called *scale effect*. Students might relate their discoveries in Activity 20 and 21 to scale effect. When a wooden beam is doubled in length, width, and thickness, its cross-sectional area is multiplied by 4 and its volume by 8. The strength of a beam depends on its cross-sectional area, but its weight depends on its volume, and thus the weight of the beam increases faster than its strength. Although wood can serve well as a joist to span a room, it is inadequate to span a larger interior space, such as a factory or cathedral, or to form a bridge over a wide lake. For such functions other materials, such as steel beams or trusses, are required. For further information, see Reid (1984).

The size and shape of a building are also determined by environmental requirements such as heating, cooling, and light. For example, heat loss occurs through exterior walls surface area. In a cold climate, a building should have the minimum surface area possible for a given volume. A sphere is the form with minimum surface area (as one can know intuitively by thinking of blowing up a balloon—the rubber contracts to minimize the surface area surrounding the given volume of air in the balloon). Students might be asked if they know of any buildings with spherical forms. Examples are igloos and certain types of bird nests. It is helpful to use an overhead transparency of Activity 22*a* to focus discussion on the properties of buildings. For further reference, see Taylor (1983) and Reid (1984, chap. 2).

Assessment: (1) Portfolio: Develop a small collection of pictures or examples of real-world buildings or objects, classify them, and discuss why they might be the shapes they are *or* make a survey of local buildings and compare the number of windows with the amount of interior space.

(2) Create some pairs of similar three-dimensional sculptures (or buildings) with cube-o-grams modeling the "scale effect"; describe your results.

Activity Sheet 22b. Be an Architect

Materials: Copy of Activity Sheet 22*b* for each student, supply of wooden cubes or sugar cubes

Buildings need light, especially in the absence or restriction of natural light. This activity has students explore the relationship between "windows" (lateral surface area) and "rooms" (volume) of buildings made with cubes. Students might realize that natural light cannot suffice for a large cubical building, again because volume increases more quickly than surface area as a given building form is enlarged. They might examine how modern skyscrapers overcome this problem (with artificial light) and how designers of early apartment buildings responded to needs for both air and light—needs often described in building codes (with air shafts). The question "How does the scale effect relate to the history of the design of cathedrals?" is described by Gould (1977). Early churches were small and simple in their floor plans. Later, larger cathedrals solved the problem of inadequate light for interior space by being relatively long and having many "outpouchings."

◆ ◆ ◆ ◆ ◆ ◆ ◆ ◆

There are further possible architectural or measurement connections that students might explore. In particular, the measurement of angles is important when considering how to design a building so that maximum light is admitted and the heat of the sun's rays is kept out. In hot climates, roof overhangs are sometimes designed so that they keep out the summer sun's midday rays but admit the cooler winter sun's rays (see fig. 20). Students might be given the angles at which summer and winter midday sun rays descend and be challenged to calculate the appropriate overhang (see Taylor [1983, p. 55]). In the days when glass was scarce or expensive and walls were thick to support the building above, windows were designed with beveled jambs to admit more light. Students might examine different possible designs for window jambs (see fig. 21) and calculate the amount of light each design admits. (See Taylor [1983, p. 143].)

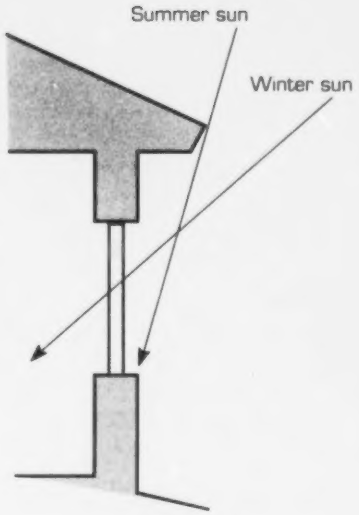


Fig. 20

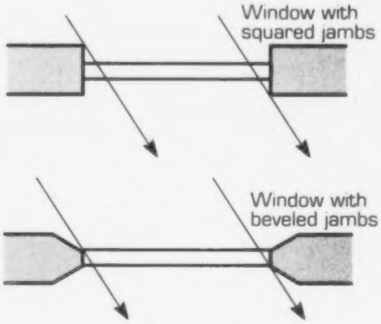


Fig. 21

Activity 22c. Building with Paper

Materials: Copy of Activity Sheet 22*c* for each student, the materials for each of the four activities listed on the activity sheet

Another excellent resource on the connections between mathematics and architecture is the work of Mario Salvadori. In his books many experiments investigating properties of materials involve mathematics while leading to the discovery of important principles of engineering (see Salvadori and Tempel [1983]; Salvadori [1990]). An example of this type of experiment is included on Activity Sheet 22*c* consisting of four activity cards that introduce some principles of construction while using skills of measurement, data collection, and the construction of solids. Students should work in small groups. Activity A is quite open. Students may come up with a variety of techniques for strengthening bridges. Activities B and C are more structured and are based on lessons 33 and 34 in Salvadori and Tempel (1983). If students repeat Activity A after doing B, C, and D, they may apply the principles learned and improve their results.

Assessment—journal entry: Describe your success rate with parts A, B, C, and D in Building with Paper

Activity 23a. Heartbeats

Materials: Copy of Activity Sheet 23*a* for every student, minute or second timer, computers and AppleWorks or MECC Create-a-Base software for data analysis (see bibliography)

Before engaging in this activity, students should have some experience with graphing, particularly, scatter plots. To answer question 4, students will need to invent a way to measure if one set of data is "closer" than another. For example, they might find the average of the data and see by what percent various numbers differ from the average for each set. This activity provides an excellent setting in which students can use a computer for data analysis.

Why are animals shaped as they are? Could a mouse be as big as an elephant or an elephant as small as a mouse? These questions can be examined in the context of the relationships among height, surface area, and volume that students discovered in Activity 20*b*. These relationships may have implications for anatomy because different biological functions depend on different aspects of an animal's form. Some of the implications of these relationships have been summarized in "On Being the Right Size" (Haldane 1956) and "Size and Shape" (Gould 1977). As Gould (p. 171) states:

> Animals are physical objects. They are shaped to their advantage by natural selection. Consequently they must assume forms best adapted to their size. The relative strength of many fundamental forces (gravity, for

> example) varies with size in a regular way, and animals respond by systematically altering their shapes. The geometry of space itself is the major reason for correlations between size and shape.

If a mouse were as big as an elephant, it would not be able to stand. The legs of a large animal must be thick because the strength of leg bones depends on their cross-sectional area. If a mouse 6 cm tall was enlarged 40 times to the height of an elephant, its weight would be multiplied by 40 × 40 × 40, or 64 000, but the strength of its leg bones would be multiplied by 40 × 40, or 1 600. Compare this idea with the ideas in Activity 20 on scale effect.

There are advantages to a mouse in being small. As Haldane (1956, p. 973) states,

> To the mouse and any smaller animal... [gravity]... presents practically no dangers. You can drop a mouse down a thousand-yard mine shaft; and, on arriving at the bottom, it gets a slight shock and walks away. A rat would probably be killed, though it can fall safely from the eleventh story of a building; a man is killed, a horse splashes. For the resistance presented to movement by the air is proportional to the surface of the moving object. Divide an animal's length, breadth, and height each by ten; its weight is reduced to a thousandth, but its surface area only to a hundredth. So the resistance to falling in the case of the small animal is relatively ten times greater than the driving force.

However, since heat is lost through an animal's skin, the food required might be expected to be proportional to surface area, not weight. A mouse should eat much more food in relation to its weight than an elephant to replace lost heat, because relative to the animal's weight, the surface area of a mouse is greater than the surface area of an elephant. In general, for a small mammal, its basal metabolic rate, the rate at which its blood circulates, and the rate at which its heartbeats are faster than those of a large mammal. However, biologists disagree about the reasons for these patterns, and there are exceptions to all of them.

The data in this activity might start students thinking about complex issues in biology. They may raise questions that will require the consultation of library resources or possibly experts in the field. First, they might think about the limits of the data. Different types of animals have different pulse rates. How should the life span of an animal be determined? Students should learn that in science even if a set of data is "clean," it can support different hypotheses, and often "right answers" are elusive, as opposed to mathematics, where ultimately all statements must stand up to the test of logic. Furthermore, mathematical models in biology seldom "fit" as closely as they do in, say, physics. Life forms have a way of finding ways around the natural "laws" that govern how they should live. However, looking at mathematical models can direct attention to why and how some organisms function in special and unusual ways.

This activity requires students to measure their own pulse rate and estimate the number of times their heart beats in a day. For background on this topic and further activities for grades 5–6 on collecting data on pulse rates, see Friel, Mokros, and Russell (1992). See also the discussion of the use of a spreadsheet in Teaching Notes for Activities 6, 13, and 16.

Assessment—journal entry: Prepare a report on an investigation you have done on pulse rates of individuals. Describe the questions you investigated, your data collection, analysis (include graphs and charts), and conjectures made.

Activity 23b. The Weight of Wetness

Materials: Copy of Activity Sheet 23*b* for each student, centimeter rulers

◆ ◆ ◆ ◆ ◆ ◆ ◆ ◆

Water clinging to a wet mouse weighs as much as the mouse itself. A wet elephant hardly notices the weight of the water. This activity challenges students to estimate the weight of water that clings to them when they are wet as a fraction of their total body weight. Then they are asked to consider the miniature children in *Honey, I Shrunk the Kids* who get wet—what fraction of their body weight are they carrying in water? For further activities relating mathematics to biology, see Mathematics Resource Project (1978).

Activity 24a. Floating Fruits

Materials: Copy of Activity Sheet 24 for each student; some oranges and other fruit; containers large enough to hold fruit and water; large pans to collect overflow of water scales; some corn syrup, alcohol, colored water; calculators

"Floating Fruits" deals with the relationship between the density of an object and its ability to float. Density is defined as the amount of mass per unit of volume of an object. The density of water is 1 gram per cubic centimeter, that is, 1 g/cc. Things that are denser than water will sink, and things less dense than water will float. One way to illustrate this concept is to take equal volumes of water, corn syrup, and alcohol and measure their weight. Corn syrup, since it is the densest, will weigh the most, whereas alcohol should be the lightest. When in a single container, the three liquids will separate out, with the corn syrup on the bottom, water in the middle, and alcohol on the top. (You may want to add some vegetable coloring to the water so it can be easily distinguished from the alcohol.)

The same is true of solids; however, their density must be calculated by dividing the weight by the volume. Although weight is easy to obtain, finding the volume of an irregular solid should be done by displacement, that is, by submerging the object in water and carefully measuring the amount of water displaced.

In this experiment, the presence of the peel on the orange affects its density, and therefore its ability to float. Since the inside of an orange is mostly water, it will sink when peeled and placed in water. However, since the peel is less dense than water, the density of the orange changes when the peel is left on, causing it to float.

The experiment is best done by students working in teams of three or four. Each person has a job, one as a "leader" who organizes the group, another as a "technician" who works with the materials, another as a "checker" who makes sure the measurements are being written down correctly, and one as a "liaison" who may ask the teacher questions only if no one in the group can help. It helps to have several oranges on hand for each group so that students can perform the experiment a few times and check for consistency. You may want to demonstrate the displacement method of finding volume so the students get accurate results and don't spill water.

This activity can be extended in several different ways by trying it with other fruits, such as bananas and grapefruit. It is also interesting to compare the density of fruits with thin peels (apples, pears, peaches, plums) with fruits having thick peels (bananas, oranges, grapefruit) or to compare the density of fruits and vegetables. Without using the method of displacement, students could find the volumes of some objects (e.g., golf or Ping-Pong balls or cubes, such as dice) by using a formula, then calculating the density and predicting whether the objects will float or sink. One could also look at different types of liquids (milk, cooking oils, juices) and predict whether they will float or sink in water. One could also bring in some chemistry, explaining that when one sees salad dressing in the grocery store, it has often separated in layers owing to the different densities of the ingredients.

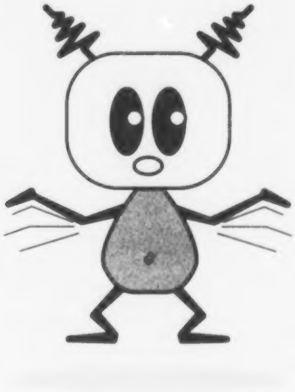
Activity 24b. Fresh vs. Salt Water and Population

Materials: Copy of Activity Sheet 24b for each student; for each group of four to six students: a graduated cylinder, 300 mL of table salt, balance, metric weights, water, paper towels, calculators

This activity gives students some experience in making predictions, designing and implementing an experiment or investigation to answer specific questions, recording, reporting, and interpreting data collected. Groups should compare their findings and obtain a class average for the density of fresh and salt water (i.e., for fresh water 1 g/cm, and for salt water 1.2 g/cm).

The second part of the activity focuses on population density and furnishes an opportunity to relate mathematics and measurement to aspects of the social studies curriculum. Students might be encouraged to extend their investigation to find the population density of several countries in the world. It is interesting and visually striking for each student to prepare two charts, each on a 10-cm-by-10-cm grid; color in the number of units of land area of one country on one chart and use the same color to record the number of units of population on the other chart, then continue the procedure for each country, recording in different colors. The population density of countries becomes visually apparent to students!

ACTIVITY 16
PLANET JUMPING



What is your track record on other planets?

Gravity is a force that pulls objects together. The more mass something has, the greater its gravitational force. For example, since the earth has a great deal of mass, it also has a great deal of gravity, which means it pulls lots of things toward it (e.g., buildings, trees, you, even the air you breathe!). You also have gravity, but since you have so little mass, you don't attract things as strongly as the earth.

In this activity you will be working with a partner to find out how you would perform in two Olympic events, the vertical jump and the long jump, on other planets.

MEASUREMENTS ON EARTH

The *vertical jump:* Stand next to a wall and stretch your arm over your head as far as possible. Mark a line where the tip of your hand touches the wall. Now jump as high as you can and leave a mark (dip finger in water before you jump) on the wall where your hand touched. Measure the distance between the two marks with your partner using a centimeter ruler. Record your measurement here: ________

The *long jump*: Mark a line on the ground, and place the tips of your feet on the edge of the line. Jump forward as far as you can. Have your partner make a mark where the heels of your feet land. Now measure the distance between the two lines. Record your measurement here: ______________

JUMPING ON THE MOON

Because the moon has only 1/6 the mass of the earth, it therefore has only 1/6 the gravitational pull. If you were competing in the Olympics on the moon, you could jump 6 times farther.

What would be your results in the vertical jump and the long jump on the moon?

Vertical jump: ______________ Long jump: ______________

Sketch a drawing of the distances you would jump on the earth compared to the distances of the jumps on the moon. Be sure to label your distances.

HOW HIGH ON NEPTUNE?

If you were competing in the Olympics on Neptune, you would be able to jump only 5/6 as far.

How does Neptune's gravity compare to Earth's gravity? What does that tell you about Neptune's mass? ____

What would be your results in the vertical jump and long jump on Neptune?

Vertical jump: ______________ Long jump: ______________

ACTIVITY 16
(CONTINUED)

MOVING AROUND ON MARS

If you were on Mars, you would be able to jump 2 3/5 times higher than on Earth.

Do you think Mars has more mass than Earth or less? Why? ______________________

____

What would be your results in the vertical jump and the long jump on Mars?

Vertical jump: ______________ Long jump: ______________

JUMPING ON JUPITER

If you were on Jupiter, you would be able to jump 5/12 the distance jumped on Earth.

Which planet has a stronger gravitational pull, Jupiter or Neptune? Why? ______________________

____

What would be your results in the vertical jump and the long jump on Jupiter?

Vertical jump: ______________ Long jump: ______________

A VISIT TO VENUS

On Venus, you would be able to jump 1 1/7 times higher than on Earth.

What would be your results in the vertical jump and the long jump on Venus?

Vertical jump: ______________ Long jump: ______________

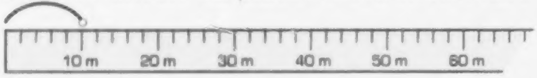
THE SUN

How would you figure out the distance you could jump on the sun (without burning your feet)?

Below is a chart showing Elvis Presley's mass on different heavenly bodies at the time of his death. Using this information, rank the mass of these bodies from least to greatest.

Planet	Elvis's Mass	Ranking
Mercury	44 kg	
Venus	105 kg	
Earth	115 kg	
Mars	44 kg	
Jupiter	294 kg	
Saturn	125 kg	
Uranus	105 kg	
Neptune	137 kg	
Pluto	6 kg	
Moon	20 kg	
Sun	3239 kg	

Below is a diagram showing the path a ball would follow if thrown 10 meters on Earth. Estimate and draw the path of a ball if thrown on the planets listed in the chart above. Explain how you arrived at your estimates.



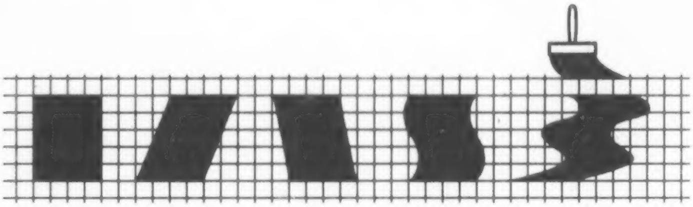
Explanation: ______________________

____

ACTIVITY 17
PAINTBRUSH MEASURE

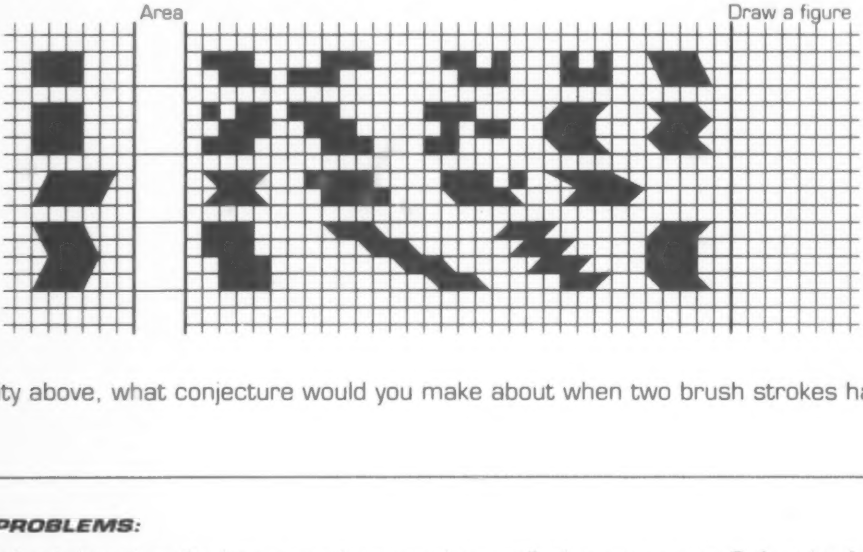
Problem: You want to cover your desktop, but you need to measure the amount of space (area) on your desktop. What unit might you use to measure this area? Explain. Could you use playing cards? A pencil? A centimeter? Tiles? What else could you use?

A painter was imagining how she could use brush strokes to cover a wall. She wondered if she would cover different amounts of area with different brush strokes. She made the following brush strokes on a wall.



Is the width of each brush stroke the same? How do you know?
Is the vertical distance of each brush stroke the same? How do you know?
Do all the brush strokes have the same area? How can you tell?
What unit are you using to measure the area? Explain.

In the chart below,
- (*a*) find the area of the first figure on each line;
- (*b*) cross out every figure on each line that does not have the same area as the first figure;
- (*c*) on each line, draw a figure of your own (in the space at the right) that has the same area as the first figure on the line.



From the activity above, what conjecture would you make about when two brush strokes have the same area?

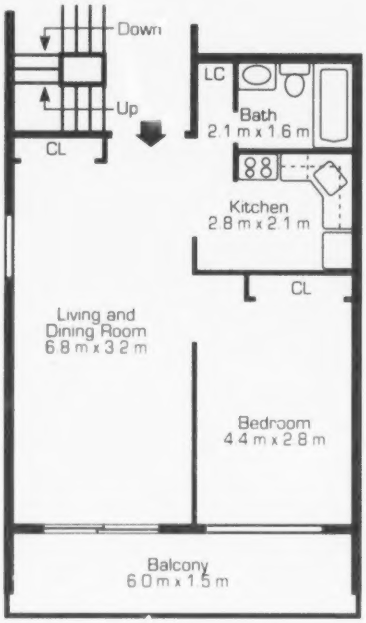
Conjecture: ____

CHALLENGE PROBLEMS:

1. A designer of patchwork quilts has an order to make a quilt that measures 2.1 m by 1.8 m with patches that are in the shape of square decimeters. If the design of the quilt requires patches of nine different colors, how many patches of each color does he need? If a fringe is used to trim the edge of the quilt, how much fringe does he need?
2. The dimensions of a room are 12 m long, 5 m wide, and 3 m high. If the floor is to be covered with square decimeter tiles, what would be the cost of the tiles if they sell for $20 per thousand tiles?

ACTIVITY 18A
SCALE MODELS: INTERPRETING A FLOOR PLAN

Examine the floor plan of the apartment at the right.



How many bedrooms are there?

How many closets are in the apartment?

How many windows are there?

The ceilings throughout the apartment are 2.6 m high.

What are the dimensions of the
- *a.* bedroom?
- *b.* living and dining room?
- *c.* kitchen?
- *d.* bathroom?
- *e.* balcony?

How many square meters are in the
- *a.* bedroom?
- *b.* living and dining room?
- *c.* kitchen?
- *d.* bathroom?
- *e.* balcony?

If you wanted to carpet the bedroom and the living and dining room, how many square meters of floor space would you tell the store you have?

If the carpeting you choose costs $30 a square meter, how much would it cost to carpet the bedroom and the living and dining room? Explain.

You have a choice of laying linoleum or tiling the floor in the kitchen. Linoleum costs $10 a square meter and tiles in the shape of square decimeters cost $50 per box of 600 tiles. If you wish to keep the cost to a minimum, which would you choose? Explain your decision.

CHALLENGE PROBLEM:

It will cost extra to wallpaper the living and dining room and the bedroom of the apartment instead of painting. However, you and your friend want to do it yourselves. Determine how much wallpaper you will need and visit a local paint and wallpaper shop to find the cost of a roll. Write in your journal how you would estimate the amount of wallpaper needed, how you determined the cost, and other questions you had to resolve.

ACTIVITY 18B
BUILDING A MODEL OF AN APARTMENT TO SCALE

Examine the floor plan of the apartment shown.

What are the length and width of each room?

Master BR: length =
width =

Bedroom 2: length =
width =

Bedroom 3: length =
width =

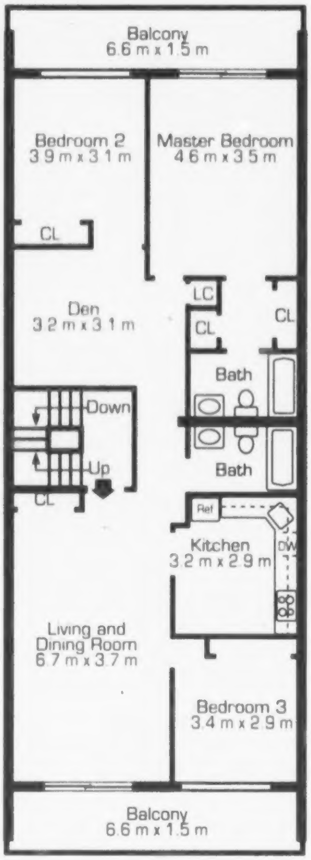
Den: length =
width =

Living/dining room:
length =
width =

Kitchen:
length =
width =

Front balcony:
length =
width =

Back balcony:
length =
width =


Balcony
6.6 m x 1.5 m
Bedroom 2
3.9 m x 3.1 m
Master Bedroom
4.6 m x 3.5 m
CL
LC
CL
CL
Den
3.2 m x 3.1 m
Bath
Down
Up
Bath
CL
Ref
Kitchen
3.2 m x 2.9 m
DW
Living and
Dining Room
6.7 m x 3.7 m
Bedroom 3
3.4 m x 2.9 m
Balcony
6.6 m x 1.5 m

How many closets are there?

How many bathrooms are there?

Are the bathrooms full or half baths?

What kind of appliances come with the apartment?

Set up a scale for the floor plan your group is working on.

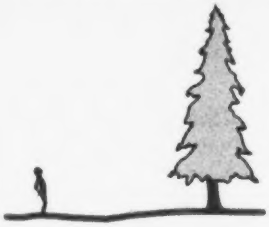
Your scale: __________ m = __________ cm

Using the materials given to you, set up a model of the floor plan your group is working on. Imagine that the contact paper furnished is the carpeting that you want to put down throughout the apartment. Cut out pieces of carpeting to fit each room.

Discuss the following questions with other members of your team. (Choose one person in your group to write down your comments to be shared with the class.)

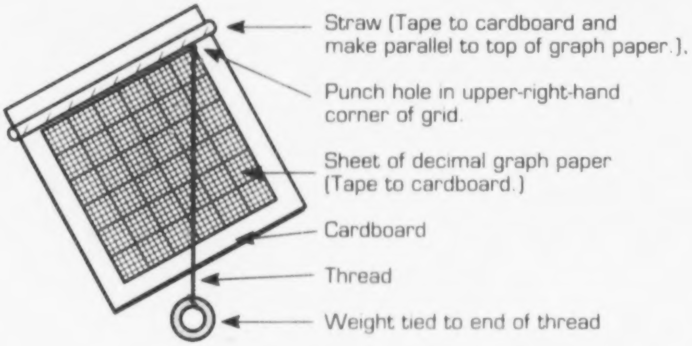
1. What percent of the apartment is carpeted?
2. What percent of the apartment is storage space?
3. What did each person in your group do toward the successful completion of this activity?
4. What did you like most about this activity? What did you like least?
5. What problems did you encounter along the way? How did your group overcome any problems?

ACTIVITY 19
FIELD MATHEMATICS—MAKING AND USING A HYPSOMETER

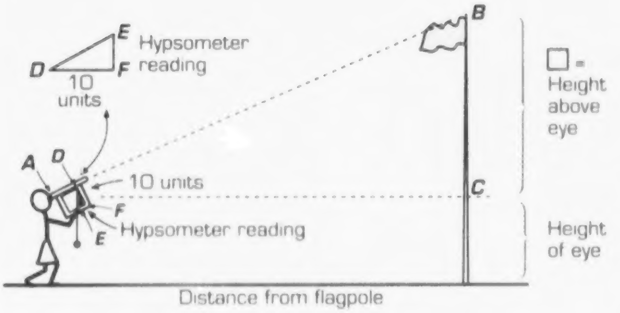


Problem: How can you find the height of a tree without climbing the tree?

An instrument used by surveyors to measure distances (particularly heights) indirectly is called a hypsometer. It is easy to make one. Here are the directions.


Straw (Tape to cardboard and make parallel to top of graph paper.).
Punch hole in upper-right-hand corner of grid.
Sheet of decimal graph paper (Tape to cardboard.)
Cardboard
Thread
Weight tied to end of thread

Let's see how you could find the height of a flagpole using the hypsometer your group made.


E
D
F
Hypsometer reading
10 units
A
D
10 units
F
E
Hypsometer reading
B
C
☐ = Height above eye
Height of eye
Distance from flagpole

Hint: Look at the small (shaded) and large triangle *(ABC)*. We can redraw the small one so the positions are the same.

How would you describe these two triangles? Justify why you think the two triangles have a special relationship to each other.

Suppose a student in your class thought the following relationship was true in the diagram:

$$\frac{\text{distance from flagpole}}{10} = \frac{\text{height above eye}}{\text{hypsometer reading}}$$

The student then used this relationship to solve the following problem:

A man is standing 20 m from the flagpole. His eye is 1.5 m above the ground. The hypsometer reading is 2.5.

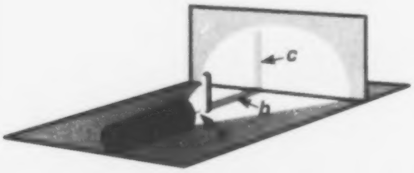
$\frac{20}{10} = \frac{\_\_\_}{2.5}$; so $2 = \frac{\_\_\_}{2.5}$ and $\quad = 5$ m

The flagpole is 5 m + 1.5 m, or 6.5 m tall.

Do you think the student is right? Why or why not?

Now it is your turn. Your group should plan a project to find the height of several objects around your school yard using your hypsometer.

ACTIVITY 20A
MEASURING SHADOWS



Question: How do you think the measurements of an object compare with the measurements of the shadow of that object projected on a wall?

Investigation: Set up a sheet of cardboard vertically on a table. Shine a flashlight onto the cardboard and place a 10-cm stick (or pencil) vertically between the light and the cardboard.

Note in the diagram the distance *a* between the light and the stick, the distance *b* between the stick and the cardboard, and the height *c* of the shadow. Measure and record these distances in the chart at the right.

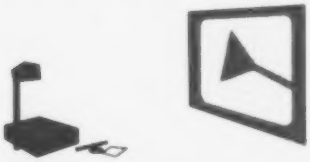
Move the cardboard *away* from the stick. Measure and record distance *b* and the height *c* of the shadow. Repeat by moving the cardboard away from the stick several times and record distances. Try changing the distance *a* and repeat the steps above. Examine the data in your chart. What conjectures can you make on the basis of the data collected?

a	*b*	*c*

Conjecture: ____

____

Let's explore further. Instead of a flashlight, let's use an overhead projector.



Investigation: Set up the projector as shown. Assemble a group of objects such as a file card, bottle cap, cut-out triangle, spoon, key. Place a transparent centimeter grid on the projector and then place each object to be measured on top of this grid. Fill in the chart below. Measure the object with a ruler and transparent grid and the shadows by counting squares on the sheet of paper on the wall.

Distance of projector from the wall = ______ .

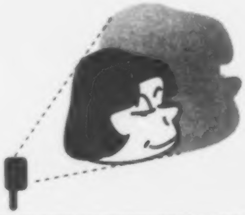
Object	Measurements of the original (to the nearest mm)				Measurements of the projection (to the nearest mm)			
	Bottom edge	Side edge	Perimeter	Area	Bottom edge	Side edge	Perimeter	Area

What can you conclude?

When the length is multiplied by _____, the width is multiplied by ____, the perimeter is multiplied by ____, and the area is multiplied by _____.

Extension: Will the results be the same if the distance between the projector and the wall is changed? Why or why not?

Challenge Problem: If a person's profile is 30 cm in height and the shadow it casts is 40 cm in height, find the ratio of the area of the profile to the area of the shadow.



ACTIVITY 20B
GRID ENLARGEMENTS AND BUILDING STAIRCASES

GRID ENLARGEMENTS

Draw enlargements of figure A on the grid. Both have been started. For each figure, find the length of the base, the perimeter, and the area.

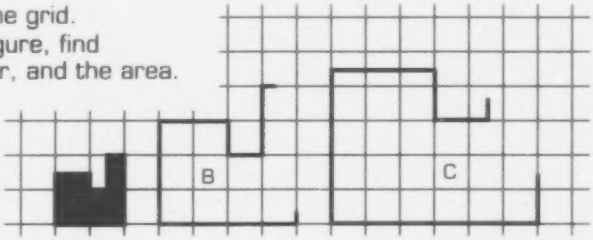


Figure A
Length = ______
Perimeter = ______
Area = ______

Figure B
Length = ______
Perimeter = ______
Area = ______

Figure C
Length = ______
Perimeter = ______
Area = ______

What do you notice?

Complete:

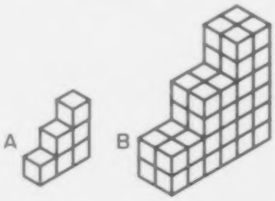
When you multiply the length by 2, you multiply the perimeter by ___ and the area by ___.
When you multiply the length by 3, you multiply the perimeter by ___ and the area by ___.
When you multiply the length by X, you multiply the perimeter by ___ and the area by ___.

Repeat this process—draw your own shape and enlarge it. Do the same patterns hold?

BUILDING STAIRCASES

Build a "staircase" like the one in figure A from wooden cubes or centicubes. Next to it build another staircase like that in figure B, which is exactly twice as large in every direction.

How do the measurements of staircases A and B compare? Complete the table. (Count the face touching the table when you measure the surface area.)



Measurement	Staircase A	Staircase B	Staircase C
Height (using the side of the cube as the unit of length)			
Surface area (using the face of the cube as the unit of area)			
Volume (using the cube as the unit of volume)			

What do you notice?

Complete: If you multiply the height by 2, you multiply the surface area by ___ and the volume by ___.

Add cubes to staircase B to make staircase C, which is exactly three times as large as staircase A in every dimension. Fill in the last column of the table above for staircase C. What do you notice?

Extension: Use Plasticine to make two spherical balls, one twice the diameter of the other. Find their volumes. (You can do this by immersing them in water and measuring the water displaced.) What is the relationship between their volumes? Find their weights. What do you notice?

ACTIVITY 21A
"LOST AND FOUND"

The cards below are each to be put in a bag with an object that is similar to the object described but is of the wrong size for a normal person—for example, a bracelet instead of a ring, a toy car, a doll's teacup, a napkin instead of a tablecloth, a large cooking spoon instead of a teaspoon, a wastebasket instead of a thimble.

Someone lost her ring.
How tall is she?

What measurements should be taken to find the height of the person who lost the object?

How did you find your answer?

Do you think that other people might have a very different answer?

Someone just got out of her car.
How tall is she?

What measurements should be taken to find the height of the person who lost the object?

How did you find your answer?

Do you think that other people might have a very different answer?

Someone lost a tablecloth for his card table.
How tall is he?

What measurements should be taken to find the height of the person who lost the object?

How did you find your answer?

Do you think that other people might have a very different answer?

Someone dropped his teaspoon.
How tall is he?

What measurements should be taken to find the height of the person who lost the object?

How did you find your answer?

Do you think that other people might have a very different answer?

Someone lost her teacup.
How tall is she?

What measurements should be taken to find the height of the person who lost the object?

How did you find your answer?

Do you think that other people might have a very different answer?

Someone lost a thimble.
How tall is he?

What measurements should be taken to find the height of the person who lost the object?

How did you find your answer?

Do you think that other people might have a very different answer?

ACTIVITY 21B
HONEY, I SHRUNK THE KIDS

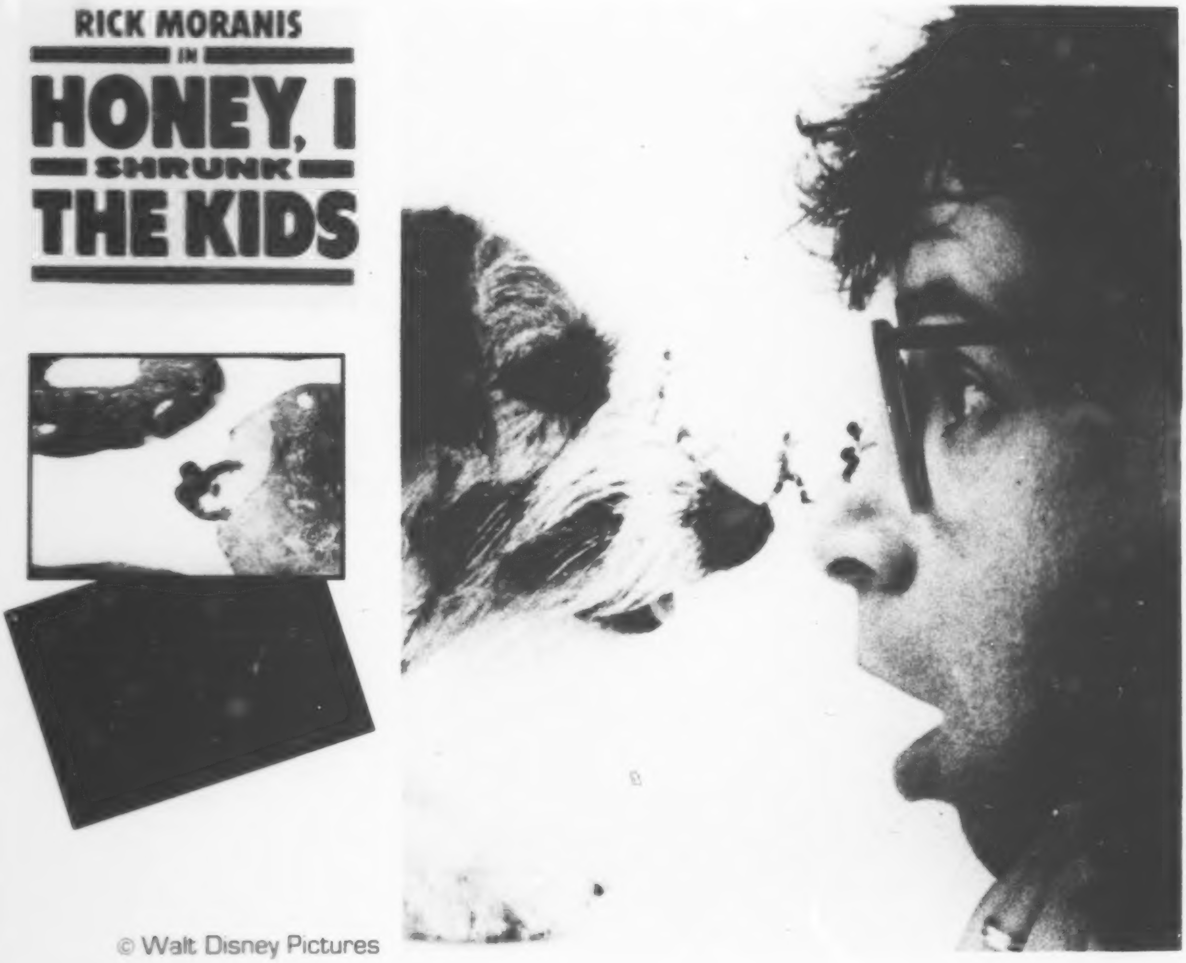
Get set for the adventure of a lifetime in the #1 comedy hit of the year, *Honey, I Shrunk the Kids!* Rick Moranis stars as the preoccupied inventor who just can't seem to get his electromagnetic shrinking machine to work. Then, when he accidentally shrinks his kids down to 1/4 inch tall and tosses them out in the trash, the real adventure begins! Now the kids face incredible dangers as they try to make their way home through the jungle of their own backyard! Hurricane sprinklers! Dive-bombing bees! A runaway lawn mower and much, much more! Directed by Academy Award–winner Joe Johnston (Visual Effects, *Raiders of the Lost Ark*, 1981), this record-breaking smash is full of amazing special effects, hilarious comedy, wild chases, and nonstop surprises!

About how tall are the shrunken children in these pictures?

How could you find out?

Are the answers the same for each picture?

Do your results agree with the statement that the kids have been shrunk to 1/4 inch?

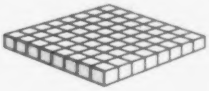


© Walt Disney Pictures

ACTIVITY 22A
THE SHAPE OF THINGS

How is the shape of a solid related to its surface area? Suppose that each of the following "buildings" has the same volume, 64 cubic units. For each, find the surface area (not counting the base) and the ratio of volume to surface area.

Rectangular Solid

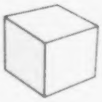


Volume = 64 units^3

Surface area = ____ units^2

Volume/surface area ratio = ____

Cube



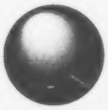
Volume = 64 units^3

Edge = ____ units

Surface area = ____ units^2

Volume/surface area ratio = ____

Sphere



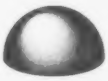
Volume = 64 units^3

Radius = ____ units

Surface area = ____ units^2

Volume/surface area ratio = ____

Hemisphere



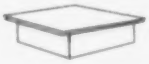
Volume = 64 units^3

Radius = ____ units

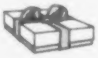
Surface area = ____ units^2

Volume/surface area ratio = ____

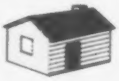
Contemporary Hawaiian house



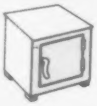
Gift candy box



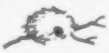
Canadian log cabin



Old-fashioned icebox

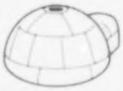


Ovenbird nest

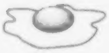


Balloon

Inuit Igloo



Yolk in a fried egg



A way to reduce a dwelling's exposure to cold is to use shapes that maximize space contained while minimizing the exposed surface areas. Explain why the buildings and objects on the right have the shape that they do.

In cold climates, why don't more cultures build in the form of spheres or hemispheres? What materials can be used to build a hemispherical building?

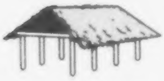
People who live in different parts of the world adapt their building traditions to match the climate, and so does nature in the form of vegetation. Match plant form to description to building below.

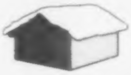
Cold—compact, closed

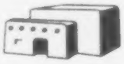
Temperate—can be more expansive

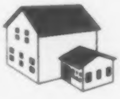
Hot, humid—expansive, shaded, ventilated

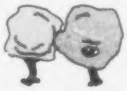
Hot, dry—compact, closed

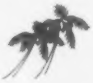


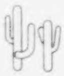






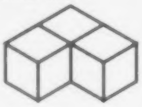






ACTIVITY 22B
BE AN ARCHITECT

You can design buildings with cubes, either wooden ones or sugar cubes. (Try to find ones that are approximately cubical.) Assume that each cube represents a "room" and that whenever a face is vertical on the outside, it has a "window." This "building" has 3 rooms and 8 windows.



Match the descriptions to the buildings below. Design a building for the extra description.

____ 6 rooms, 14 windows

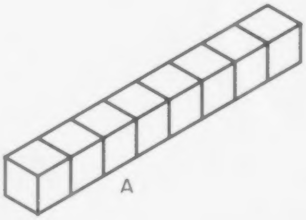
____ 6 rooms, 16 windows

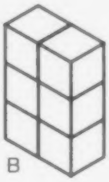
____ 6 rooms, 18 windows

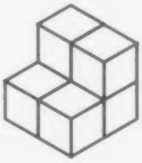
____ 8 rooms, 16 windows

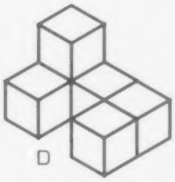
____ 8 rooms, 18 windows

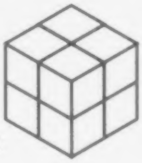
____ 8 rooms, 22 windows


A


B


C


D


E

Draw the missing building:

Can you make a building with more rooms than windows? Could this building have more than 10 windows? Fewer than 10 windows?

Why do buildings need windows?

Building codes sometimes require a certain number of windows for a given floor area or number of rooms. Suppose that you are working under a building code that requires at least one window for every two rooms. What sort of buildings could be made?

Could such a building code be applied to a large modern skyscraper?

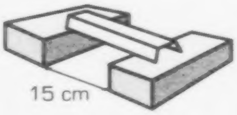
Why might a code like this not be necessary for a modern building?

ACTIVITY 22C
BUILDING WITH PAPER

A. A Paper Bridge

Materials: A piece of paper, 22 cm × 28 cm, scissors, glue, two bricks or large wooden blocks, centicubes or other units for mass

Directions:



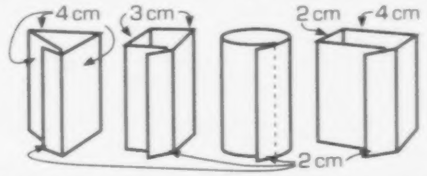
1. Build a bridge out of paper to go between the two blocks, placed 15 cm apart, so that
   a. it will hold as much weight as possible, and
   b. it is at least 5 cm wide.
2. When your bridge is completed, sketch it (including measurements) and write a description of how you made it.
3. Test how many grams your bridge will hold. Load weights onto it until it buckles. Record your results.

C. Columns

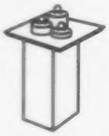
Materials: Pieces of paper, 14 cm × 14 cm, scissors, glue, units of mass (50 g to 1000 g), small square of cardboard

Directions:

1. Use strips to make columns with different cross sections: equilateral triangle, square, circle, rectangle (2 cm × 4 cm), as shown below. Allow a 2-cm strip for gluing.



2. Test to see how much weight each column will bear. Place a square of cardboard on top and load weights on it until the column gives way. Record the weight for each column.

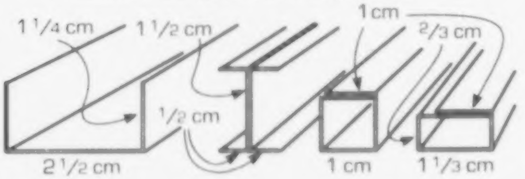


B. Beams

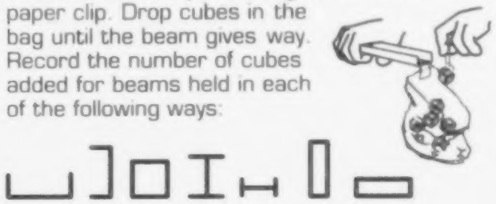
Materials: 6 strips of paper, 5 cm × 28 cm, scissors, glue, centicubes or other units for mass, paper clip, small plastic bag

Directions:

1. Measure, then fold strips of paper to make the "beams" shown below. Make 1 "C," 2 "I" beams, 1 "box," and 2 rectangles." (For the "I" beam, cut a strip in half the long way, make 2 "C" beams, and glue them together.)



2. Test the stiffness of each beam. Hold it at one end and fasten a plastic bag at the other with a paper clip. Drop cubes in the bag until the beam gives way. Record the number of cubes added for beams held in each of the following ways:

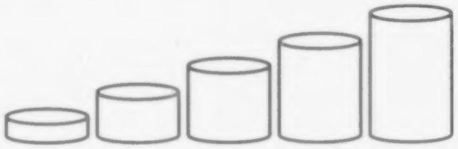


D. Comparing Results

Materials: Same as for Activity C; results from at least three other groups on Activities B and C

Directions:

1. Collect data from at least three other groups. Did you agree on which beam and column are strongest?
2. Find averages of the weight held for the beams and columns. Do the averages agree with your results?
3. Does height affect the strength of the column? Make cylindrical columns out of strips of paper 14 cm wide, and 2, 5, 8, 11, and 14 cm long. Find the maximum weight each will carry. Make a graph. Is there a pattern? Can you predict the weight that will be carried by a cylindrical column made from a strip 17 cm long?



ACTIVITY 23A
HEARTBEATS

1. Examine this chart. Do you see any patterns?

Animal	Pulse rate (number of heart beats per minute)	Weight (in kg)	Lifespan (in years)
Bat	588	0.03	20
Squirrel	390	0.5	5–12
Chicken	320	3	6–10
Porcupine	300	9	5–10
Fox	240	4	10–15
Cat	130	6	14–20
Collie	100	29	12
Sheep	75	110	15–20
Cow	55	660	15–20
Horse	35	480	20–35
Elephant	30	6100	60

Make a scatter plot, showing the weight of an animal versus the pulse rate.

Do you notice any patterns? Do larger animals seem to have faster or slower pulse rates?

The heart beats to distribute nutrient-filled blood throughout the body to replace heat lost through the skin. Do large animals have more or less energy lost through their skin in proportion to their weight? ______ Explain.

How might you explain why larger animals' hearts beat more slowly than smaller animals'?

2. On the basis of the graph and your own weight, what would you predict your pulse rate to be? Find it. Insert yourself in the graph.

3. The author Roald Dahl refers to animals' pulse rates in two of his books. In *Witches*, a grandmother tells her grandson-turned-mouse that now his mouse heart is beating at the rate of nearly 9 beats every second. However, in *Matilda* the girl genius says that the heart of a mouse beats at the rate of 650 times a second.

   Could both of these statements be correct?

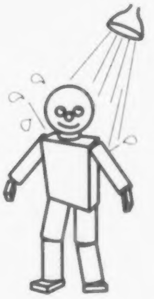
   From the information in the chart, which do you think is more likely to be true?

   In the book, Matilda also claims that a horse's heart beats 40 times a minute. Why might that figure be different from the one in the chart?

4. How many times does each animal's heart beat in a lifetime? Are these figures closer to each other than their pulse rates are? Can you explain?

Extensions: What affects your heartbeat? Is it always the same? How much variation is there among individuals?

ACTIVITY 23B
THE WEIGHT OF WETNESS



1. How much water do you think clings to you when you step out of a bath or shower? A film of water about 0.05 cm, or 0.5 mm, thick clings to our skin when we step out of the bath. How much water you carry depends on your surface area.

   Find a way to estimate your surface area in square centimeters.. It might help to sketch a very rough approximation of a person made from cylinders, rectangular prisms, and spheres, as shown.

   Your surface area is about _____ cm^2. You carry a volume of water approximately 0.05 times your surface area.

   The volume of the water that clings to your skin = _____ cm^3.

   The weight of the water that clings to your skin = ______ kg = _____ g.

   What is your weight? ______ kg = _____g.

   What percent of your body weight is the weight of water that clings to your skin? ___

   Do you notice this weight when you are wet?

2. For an average adult weighing 78 kg and with a surface area of about 9675 cm^2, this percent is about 0.6. Is that consistent with your calculations?

3. A fly weighs about 0.01 g and has a surface area of about 0.65 cm^2.

   What is the weight of the water that clings to a wet fly? _______ g

   What percent of the fly's weight is this? ___

   Could you lift a weight that was this percent of your own weight?

4. In Activity 21*a* you saw that if the height of a form is shrunk by a factor of 100, the surface area is shrunk by a factor of ________ and the weight is shrunk by a factor of _________.

   Suppose that you were shrunk by a factor of 100.

   Your new surface area = _________ cm^2.
   The volume of the water that clings to your skin = _________ cm^3.
   The weight of the water that clings to your skin = _______ kg = _______ g.
   Note: It would still be 0.05 cm thick.

   What is your new weight? ________ kg = ________ g

   What percent of your new body weight would be the weight of the water that clings to your skin? _____ Would you notice this weight?

5. In the movie *Honey, I Shrunk the Kids*, the children who are shrunk to a height of about 1 cm fall in the water. What would this feel like to them?

91

ACTIVITY 24A
FLOATING FRUITS

Questions: How dense is an orange? Will an orange float or sink in water?

Density is the mass (weight) of an object divided by the volume of the object. Water has a density of 1 gram per cubic centimeter (1 g/cc) because if we measure out one cubic centimeter of water, it weighs one gram.

Experiment: Materials needed: some oranges, a container large enough to hold an orange, a large pan, a measuring cup, a scale, a calculator

Here's what to do:

1. Measure the weight of an orange.
2. Fill the container with water and place it in the pan. Submerge the orange in the container, pushing gently on the orange to make sure it is fully under water.
3. Carefully measure the water that spilled out of the container into the pan.
4. Find the volume of the orange.
5. Find the density of the orange. (Recall the definition of density.) Try several oranges and record your results on the chart below.
6. Check to see if the oranges will float. Record your results.
7. Now peel the oranges and repeat steps 1–6 for each orange. Record your results.

Floating Fruits Record Sheet

Orange number	Orange *with* peel				Orange *without* peel			
	Weight	Volume	Density	Float?	Weight	Volume	Density	Float?

Questions about floating fruits:

1. How does the density of the peeled oranges compare to the density of the unpeeled oranges?
2. Which were denser than water, the peeled or the unpeeled oranges?
3. Is there a connection between the density of an orange and whether or not it floats? If so, describe the connection.

Extension: Do you think this experiment will work for other "peelable" fruits, such as bananas, grapefruits, lemons? Justify your conclusion. Do you think fruits and vegetables with thin peels are more likely or less likely to float than those with thick peels? Explain.

ACTIVITY 24B
FRESH VS. SALT WATER AND POPULATION

Question: Do fresh water and salt water have the same density?

What prediction(s) would you make about their density?
Prediction(s): ______________________________

Let's design an experiment to find out.
Density is defined as mass (weight) per unit of volume.

What fluids and data must be collected in order to answer the question above about fresh water and salt water? Make a list. ______________

____

____

How shall we proceed?
Experiment: Your group will need a graduated cylinder, 300 mL of table salt, a balance and weights, tap water, paper towels, calculators.

Steps to consider:

1. Preparation of salt water: Add salt a little at a time to the tap water, stirring to dissolve. Stop when the salt is no longer dissolving. It is a supersaturated solution of salt and water. Optional: add a drop of food coloring.
2. Amounts of fresh water and salt water to use
3. Weighing: the empty graduated cylinder, selected amounts of fresh water and salt water
4. What calculations are needed?
5. Should the experiment be repeated with different amounts of water?
6. Creation of chart in which to record the data collected and density calculated
7. Statement of findings and conclusions; comparison of findings with your predictions

Complete steps 1–7 above and prepare a report on your experiment.
Question: How does the population density of your state (city) compare with the population density of an adjoining state (city)?

What prediction(s) would you make?
Prediction(s): ______________________________

Recall that density is defined as mass per unit of volume.
Population density is frequently thought of as the average number of people per square kilometer of land area. For example:

State	Population	Land area (rounded to thousands of km^2)
New York	17 558 165	123 000
Connecticut	3 107 564	13 000
Pennsylvania	11 864 720	116 000

Calculate the population density for each state. Which state has the greatest population density? The least population density? Justify your conclusions.

Now it is your turn. What information is needed to find the population density of your state (city) in comparison to other geographic areas? Complete your investigation and prepare a report of your findings.

BIBLIOGRAPHY

Apostol, Tom. *The Theorem of Pythagoras.* Videotape and guide. Reston, Va.: National Council of Teachers of Mathematics, 1988.

Artzt, Alice F., and Claire M. Newman. *How to Use Cooperative Learning in the Mathematics Class.* Reston, Va.: National Council of Teachers of Mathematics, 1990.

Baroody, Arthur J. "One Point of View: Manipulatives Don't Come with Guarantees." *Arithmetic Teacher* 37 (October 1989): 4–5.

Bright, George W. "Data Bases in the Teaching of Elementary School Mathematics." *Arithmetic Teacher* 37 (September 1989): 38–42.

_____. "Estimation as Part of Learning to Measure." In *Measurement in School Mathematics*, 1976 Yearbook of the National Council of Teachers of Mathematics, edited by Doyal Nelson, pp. 87–104. Reston, Va.: The Council, 1976.

Budin, Howard, Diane S. Kendall, and James Lengel. *Using Computers in the Social Studies.* New York: Teachers College Press, 1986.

Coburn, Terrence G., and Albert P. Shulte. "Estimation in Measurement." In *Estimation and Mental Computation*, 1986 Yearbook of the National Council of Teachers of Mathematics, edited by Harold L. Schoen, pp. 195–203. Reston, Va.: The Council, 1986.

Curcio, Frances R., and Nadine Bezuk. *Understanding Rational Numbers and Proportions. Curriculum and Evaluation Standards for School Mathematics* Addenda Series, Grades 5–8. Reston, Va.: National Council of Teachers of Mathematics, forthcoming.

Dahl, Roald. *The Witches.* New York: Farrar Straus Giroux, 1983.

_____. *Matilda.* New York: Viking Penguin, 1988.

Davidson, Neil, ed. *Cooperative Learning in Mathematics.* Menlo Park, Calif.: Addison-Wesley Publishing Co., 1989.

Fay, Nancy, and Catharine Tsairides. "Metric Mall." *Arithmetic Teacher* 37 (September 1989): 6–11.

Friel, Susan N., J. R. Mokros, and S. J. Russell. *Used Numbers: Statistics—Middles, Means and In-Betweens.* Palo Alto, Calif.: Dale Seymour Publications, 1992.

Fuys, David, Dorothy Geddes, and Rosamond Tischler. *The van Hiele Model of Thinking in Geometry among Adolescents. Journal for Research in Mathematics Education* Monograph No. 3. Reston, Va.: National Council of Teachers of Mathematics, 1988.

Geddes, Dorothy, et al. *Geometry in the Middle Grades. Curriculum and Evaluation Standards for School Mathematics* Addenda Series, Grades 5–8. Reston, Va.: National Council of Teachers of Mathematics, 1992.

Gould, Stephen Jay. "Size and Shape." In *Ever Since Darwin, Reflections in Natural History.* New York: Museum of Natural History, 1977.

Haldane, J. B. S. "On Being the Right Size." In *The World of Mathematics*, vol. 2, edited by James R. Newman, pp. 952–57. New York: Simon & Schuster, 1956.

Hirsch, Christian R., and Glenda Lappan. "Transition to High School Mathematics." *Mathematics Teacher* 82 (November 1989): 614–18.

Hunt, John D. "How High Is a Flagpole?" *Arithmetic Teacher* 25 (February 1978): 42–43.

Kroll, Diana Lambdin, and Frank K. Lester, Jr. "Appendix 1: Evaluation." In *Developing Number Sense in the Middle Grades*, by Barbara Reys, pp. 50–54. Reston, Va.: National Council of Teachers of Mathematics, 1991.

Lindquist, Mary Montgomery. "Estimation and Mental Computation: Measurement." *Arithmetic Teacher* 34 (January 1987): 16–17.

Lindquist, Mary Montgomery, ed. *Results from the Fourth Mathematics Assessment of the National Assessment of Educational Progress.* Reston, Va.: National Council of Teachers of Mathematics, 1989.

Maletsky, Evan M., ed. *Teaching with Student Math Notes.* Reston, Va.: National Council of Teachers of Mathematics, 1987.

Mathematics Resource Project. *Mathematics in Science and Society.* Palo Alto, Calif.: Creative Publications, 1978.

National Council of Supervisors of Mathematics. "Essential Mathematics for the Twenty-first Century." *Mathematics Teacher* 82 (September 1989): 470–74.

National Council of Teachers of Mathematics. *An Agenda for Action.* Reston, Va.: The Council, 1980.

____. *Curriculum and Evaluation Standards for School Mathematics*. Reston, Va.: The Council, 1989.

____. *Curriculum and Evaluation Standards for School Mathematics* Addenda Series, Grades K–6, edited by Miriam A. Leiva. Reston, Va.: The Council, 1991–93.

____. *Curriculum and Evaluation Standards for School Mathematics* Addenda Series, Grades 5–8, edited by Frances R. Curcio. Reston, Va.: The Council, 1991–94.

____. *Curriculum and Evaluation Standards for School Mathematics* Addenda Series, Grades 9–12, edited by Christian R. Hirsch. Reston, Va.: The Council, 1991–93.

____. *Estimation and Mental Computation*. 1986 Yearbook of the National Council of Teachers of Mathematics, edited by Harold L. Schoen. Reston, Va.: The Council, 1986.

____. *Measurement in School Mathematics*. 1976 Yearbook of the National Council of Teachers of Mathematics, edited by Doyal Nelson. Reston, Va.: The Council, 1976.

____. *New Directions in Elementary School Mathematics*. 1989 Yearbook of the National Council of Teachers of Mathematics, edited by Paul R. Trafton. Reston, Va.: The Council, 1989.

____. *Professional Standards for Teaching Mathematics*. Reston, Va.: The Council, 1991.

National Research Council. *Everybody Counts*. Washington, D.C.: National Academy Press, 1989.

____. *Counting on You*. Washington, D.C.: National Academy Press, 1991.

Osborne, Alan. "Measurement: How Much?" In *Selected Issues in Mathematics Education*, edited by Mary Montgomery Lindquist, pp. 54–68. Chicago: National Society for the Study of Education, 1980.

Phillips, Elizabeth, et al. *Patterns and Functions*. *Curriculum and Evaluation Standards for School Mathematics* Addenda Series, Grades 5–8. Reston, Va.: National Council of Teachers of Mathematics, 1991.

Polya, George. *How to Solve It*. 2d ed. Princeton, N. J.: Princeton University Press, 1973.

Reid, Esmond. *Understanding Buildings, a Multidisciplinary Approach*. Cambridge, Mass.: MIT Press, 1984.

Reys, Barbara, et al. *Developing Number Sense in the Middle Grades*. *Curriculum and Evaluation Standards for School Mathematics* Addenda Series, Grades 5–8. Reston, Va.: National Council of Teachers of Mathematics, 1991.

Rowan, Thomas. Report of the Task Force on Addenda to the NCTM K–12 *Curriculum and Evaluation Standards for School Mathematics*. Unpublished report. Reston, Va.: National Council of Teachers of Mathematics, November 1988.

Salvadori, Mario. *The Art of Construction, Projects and Principles for Beginning Engineers and Architects*. Chicago: Chicago Review Press, 1990.

Salvadori, Mario, and Michael Tempel. *Architecture and Engineering, an Illustrated Teacher's Guide to Why Buildings Stand Up*. New York: New York Academy of Sciences,1983.

Shaw, Jean M., and Mary Jo Puckett Cliatt. "Developing Measurement Sense." In *New Directions for Elementary School Mathematics*, 1989 Yearbook of the National Council of Teachers of Mathematics, edited by Paul R. Trafton, pp. 149–55. Reston, Va.: The Council, 1989.

Shroyer, Janet, and William Fitzgerald. *Mouse and Elephant: Measuring Growth*. Middle Grades Mathematics Project. Menlo Park, Calif.: Addison-Wesley Publishing Co., 1986.

Steen, Lynn Arthur. "Teaching Mathematics for Tomorrow's World." *Educational Leadership* 47 (September 1989):18–22.

Taylor, John S. *Common-sense Architecture*. New York: W. W. Norton, 1983.

Vygotsky, Lev. *Thought and Language*. Cambridge, Mass.: MIT Press, 1934/1986.

Wilson, Patricia S., and Verna M. Adams. "A Dynamic Way to Teach Angle and Angle Measure." *Arithmetic Teacher* 39 (January 1992): 6–13.

Wilson, Patricia S., and Alan Osborne. "Foundational Ideas in Teaching about Measure." In *Teaching Mathematics in Grades K–8*, 2d ed., edited by Thomas R. Post, pp. 89–121. Newton, Mass.: Allyn & Bacon, 1992.

Zawojewski, Judith, et al. *Dealing with Data and Chance*. *Curriculum and Evaluation Standards for School Mathematics* Addenda Series, Grades 5–8. Reston, Va.: National Council of Teachers of Mathematics, 1991.

COMPUTER SOFTWARE

Geometric Investigation Software

Elastic Lines. Sunburst Communications, 101 Castleton Street, Pleasantville, NY 10570.

Euclid's Toolbox. Addison-Wesley Publishing Company, Route128, Reading, Mass.

The Geometer's Sketchpad. Key Curriculum Press, 2512 Martin Luther King, Jr., Way, P.O. Box 2304, Berkeley, CA 94702.

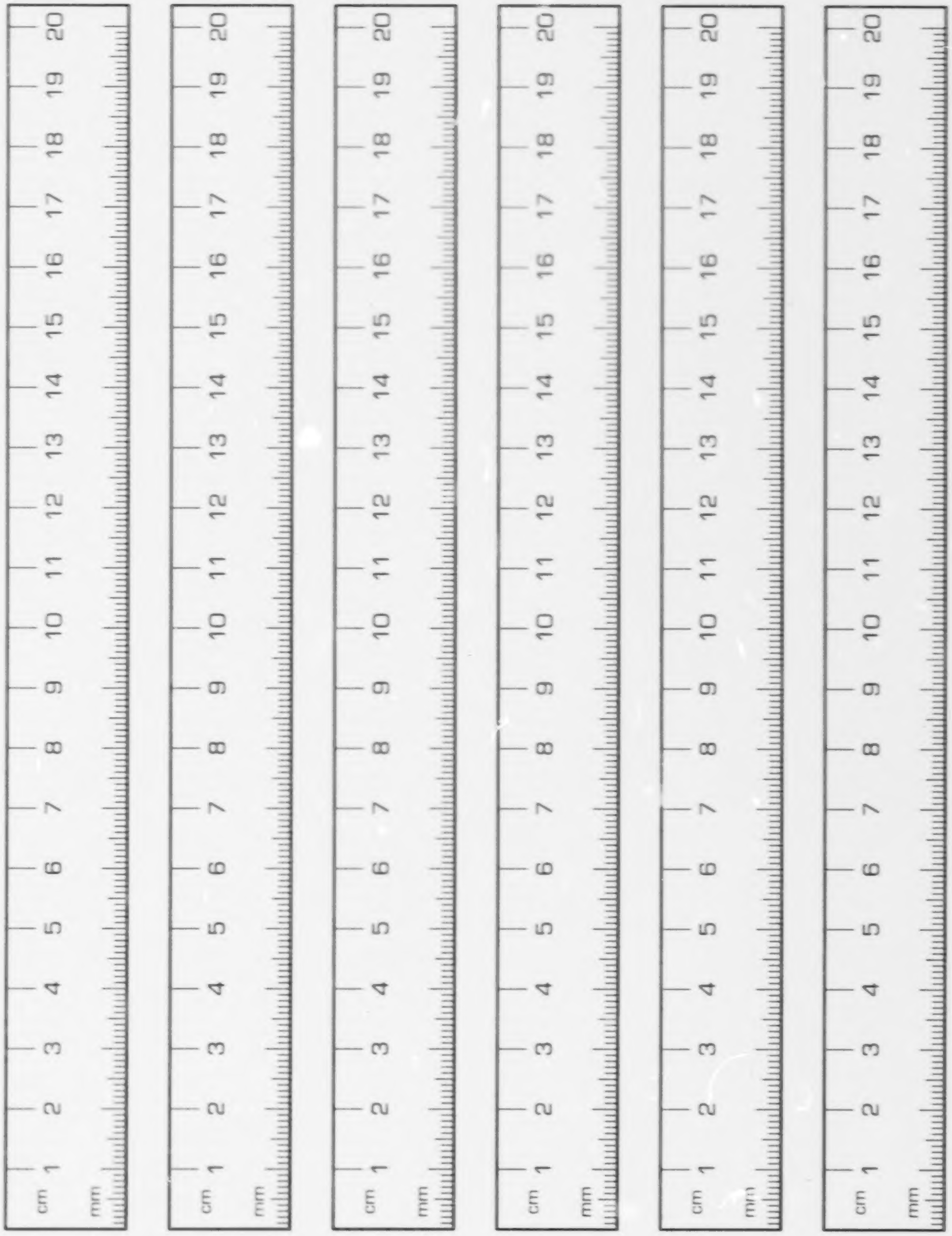
The Geometric Supposer. Sunburst Communications, 101 Castleton Street, Pleasantville, NY 10570.

Database Software

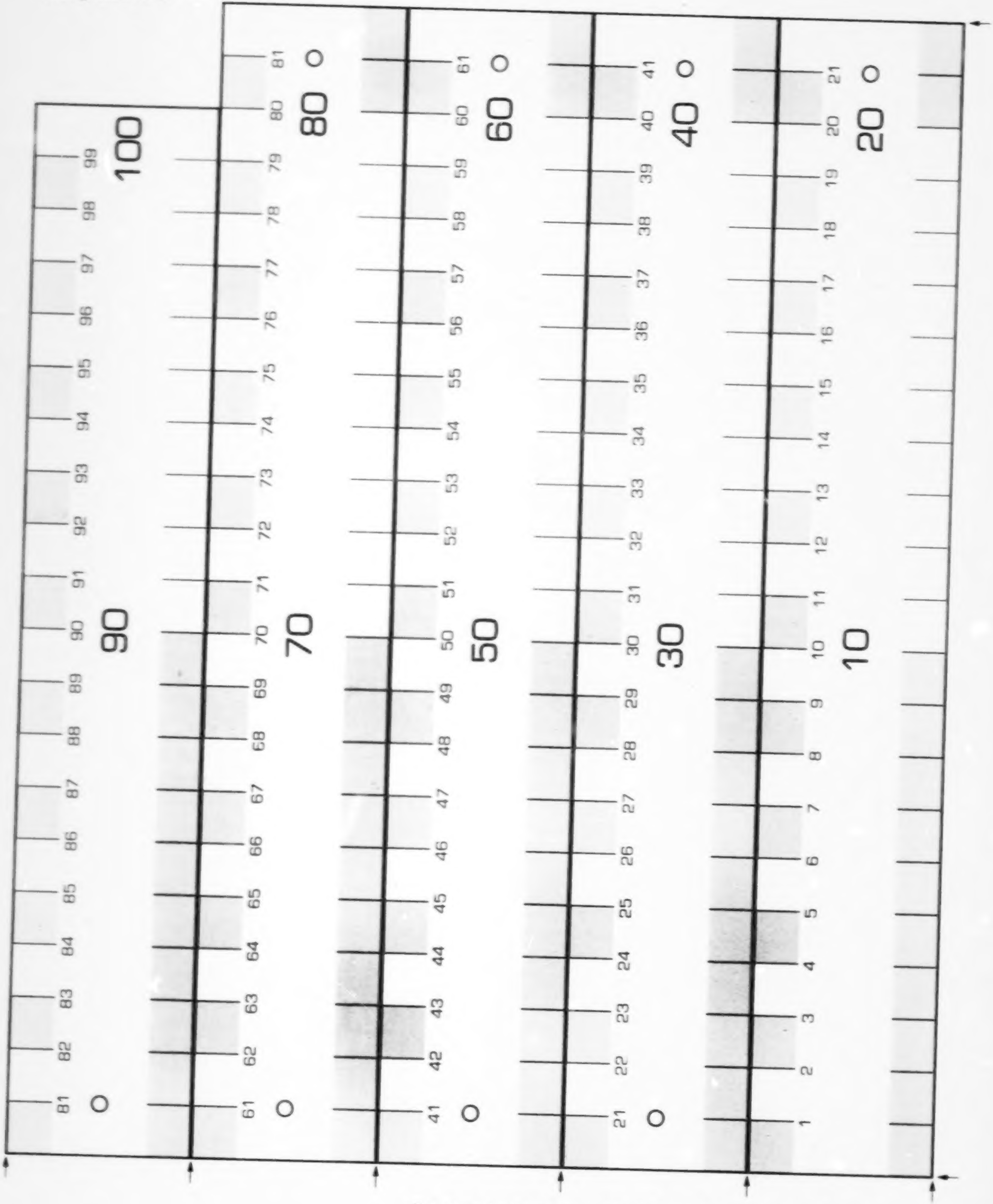
AppleWorks. Apple Computer, 20525 Mariani Avenue, Cupertino, CA 95014.

MECC Create-a-Base. MECC, 3490 Lexington Avenue North, Saint Paul, MN 55112.

APPENDIX
20-cm Rulers


cm
mm
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20

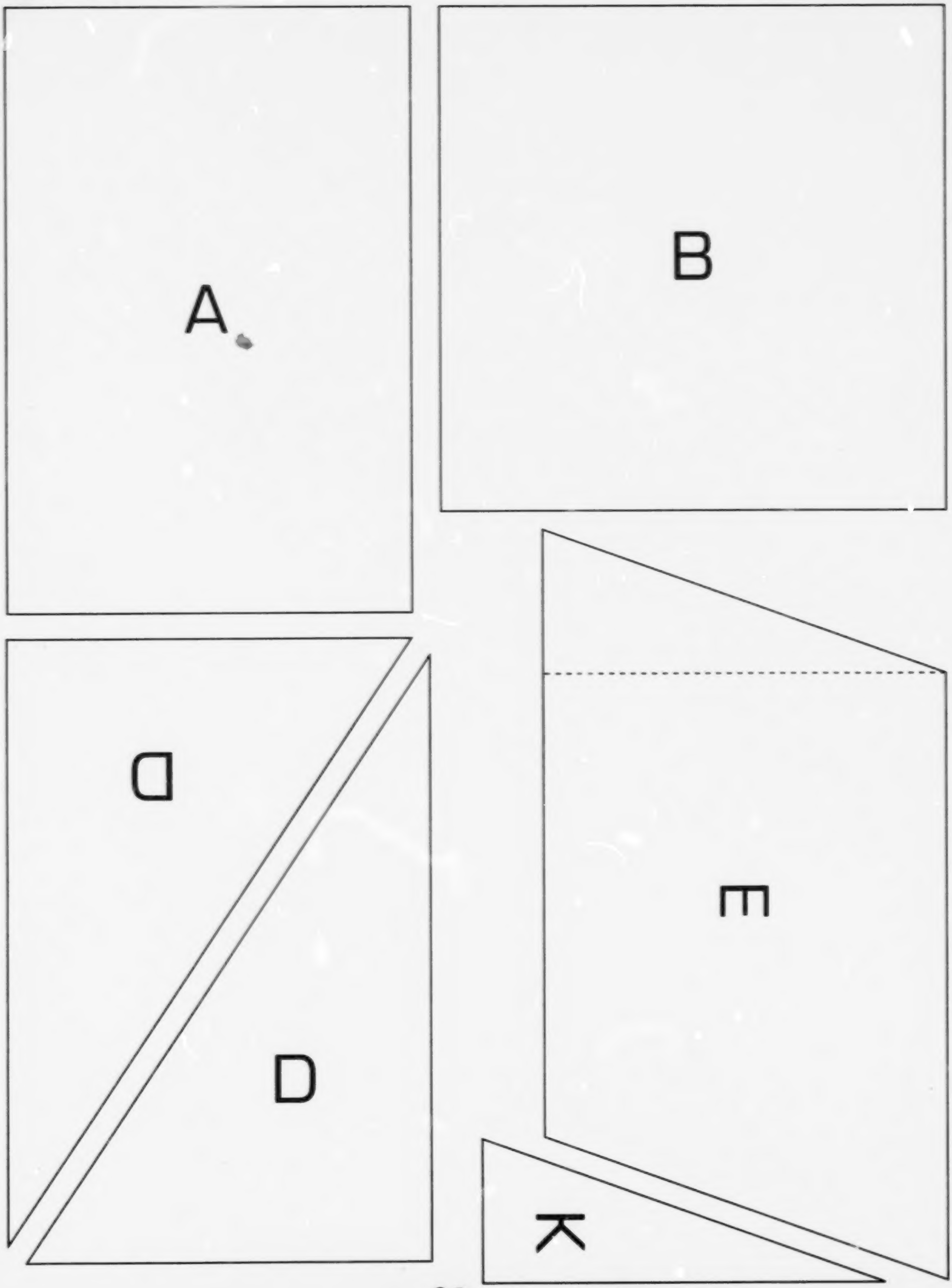
BEST COPY AVAILABLE


10
20
30
40
50
60
70
80
90
100

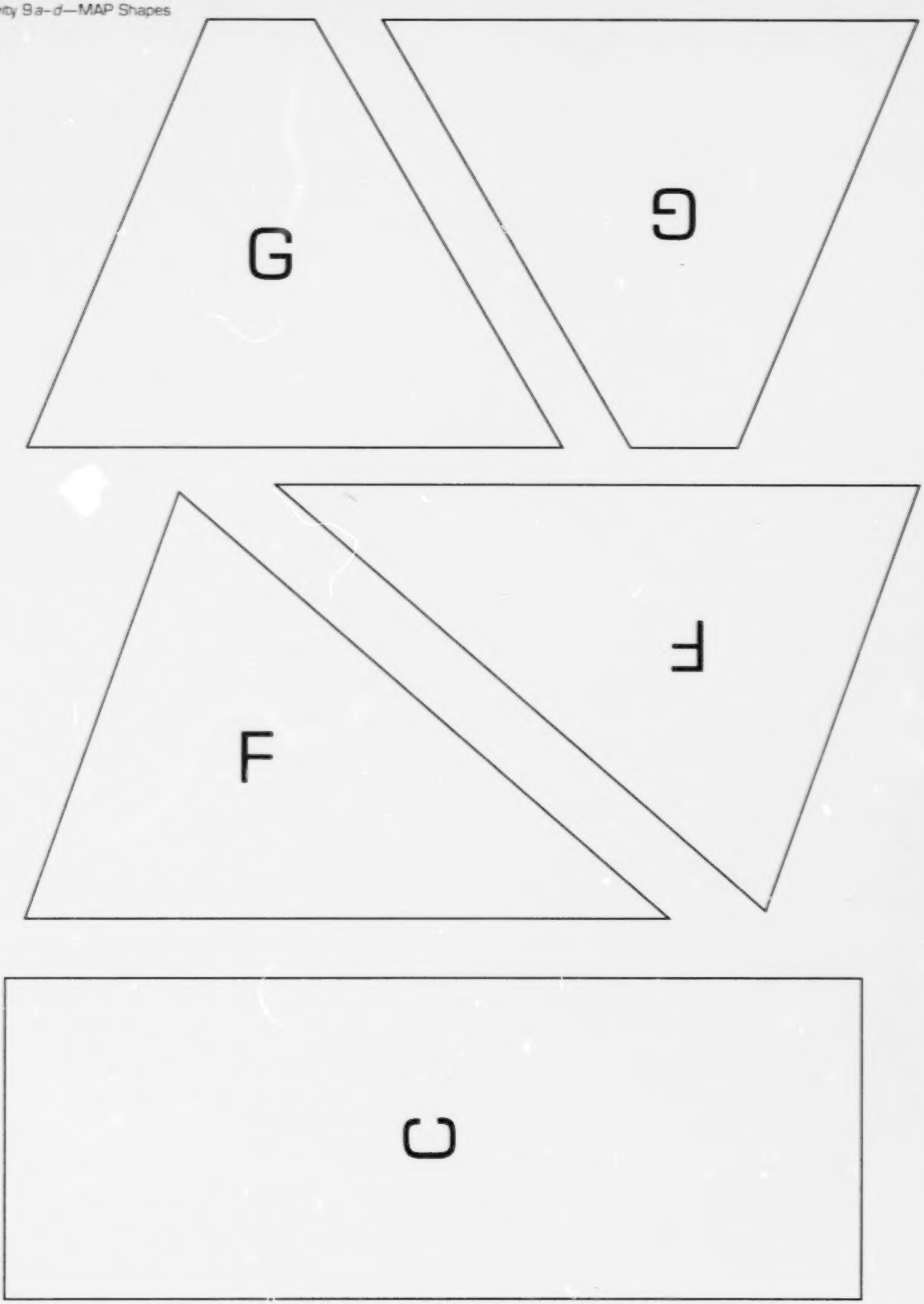
Cut along the lines indicated by the arrows. Punch holes where indicated. Assemble with paper fasteners or a dressmaker's tool for making rivets.

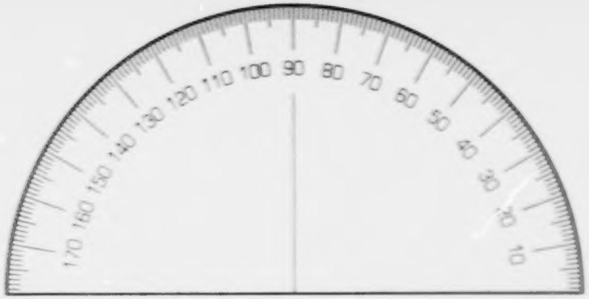
BEST COPY AVAILABLE

APPENDIX
Activity 9*a*–*d*—MAP Shapes

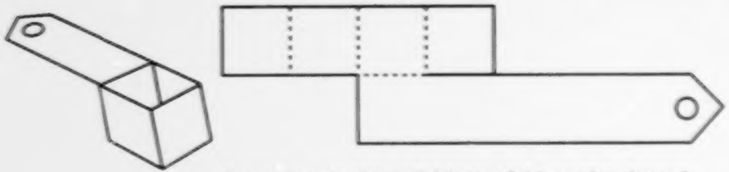


APPENDIX
Activity 9*a*–*d*—MAP Shapes

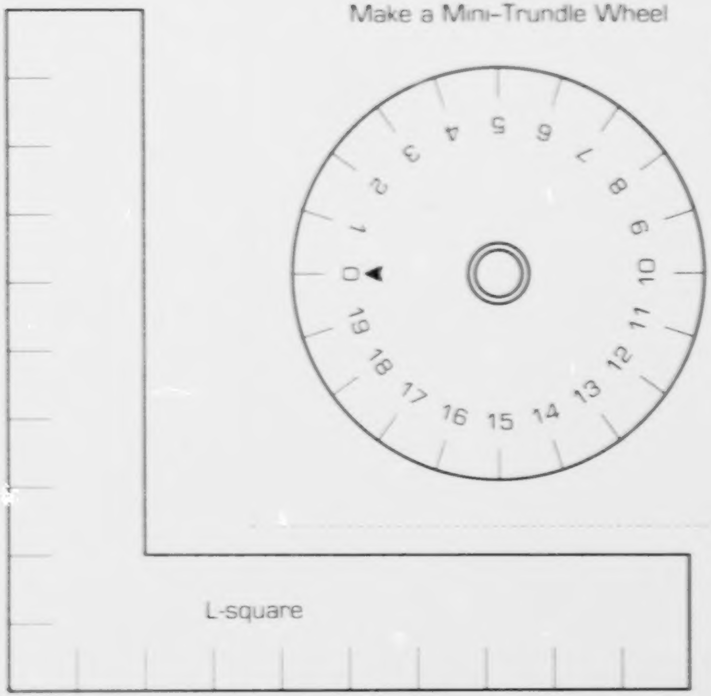



10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170

Make a Milliliter Measure



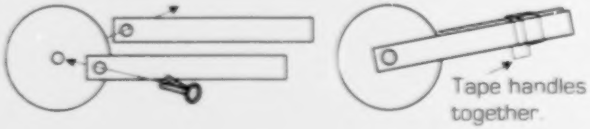
If you cut on the solid lines, fold on the dotted lines, and then tape, you can make a little square-sided spoon.


L-square

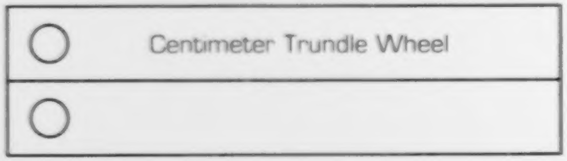
Make a Mini-Trundle Wheel

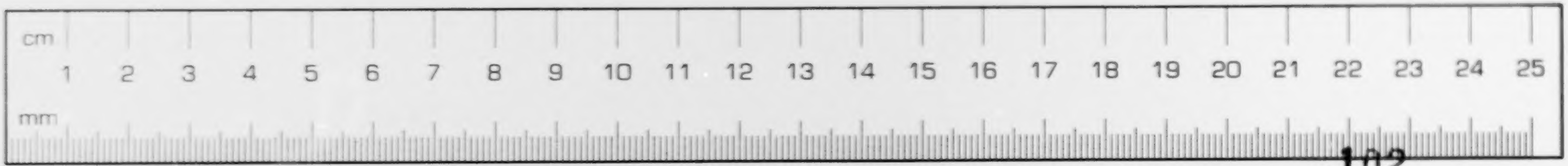
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19

Directions: Cut out the "wheel." For extra stiffness, first glue it to cardboard. Punch a hole in the middle. Cut out the two strips and punch holes at the ends of the strips. Fasten your "mini-trundle wheel" together with a paper fastener as shown below.



Tape handles together.


Centimeter Trundle Wheel


cm
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
mm

Curriculum and Evaluation Standards for School Mathematics Addenda Series

Bonnie H. Litwiller, Addenda Series Project Coordinator

Grades K–6 Addenda Series

Miriam A. Leiva, Series Editor

Seven grade-level books,
kindergarten through sixth grade
GEOMETRY AND SPATIAL SENSE
MAKING SENSE OF DATA
NUMBER SENSE AND OPERATIONS
PATTERNS

Grades 5–8 Addenda Series

Frances R. Curcio, Series Editor

DEALING WITH DATA AND CHANCE
DEVELOPING NUMBER SENSE IN THE MIDDLE GRADES
GEOMETRY IN THE MIDDLE GRADES
MEASUREMENT IN THE MIDDLE GRADES
PATTERNS AND FUNCTIONS
UNDERSTANDING RATIONAL NUMBERS AND PROPORTIONS

Grades 9–12 Addenda Series

Christian R. Hirsch, Series Editor

ALGEBRA IN A TECHNOLOGICAL WORLD
CONNECTING MATHEMATICS
A CORE CURRICULUM: MAKING MATHEMATICS COUNT FOR EVERYONE
DATA ANALYSIS AND STATISTICS ACROSS THE CURRICULUM
GEOMETRY FROM MULTIPLE PERSPECTIVES